Aircraft deconfliction via Mathematical Programming

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Abstract

Conflict detection and resolution is a core task in air traffic management. It is typically carried out manually by air traffic controllers, who are in charge of ensuring a minimum separation between aircraft during the flight. Significant effort has been invested towards automation of this task, which interest and importance is now increasing with the growth of flying vehicles. In this work, we propose an algorithm to obtain conflict-free trajectories based on adjustments of nominal velocities and heading angles. The corresponding mathematical programming formulation is nonlinear due to the intrinsic bounds on aircraft maneuvers. Our main contribution is an exact algorithm to solve the problem to optimality without relying on space/time discretization or other simplifications.

Keywords : *aircraft separation, nonlinear programming, convex relaxation, branch and cut*

1 Introduction

The number of flying vehicles is expected to grow in the near future, especially due to the development of technology and urban air mobility. Another factor contributing to this is the introduction of unmanned aerial vehicles, which are becoming more and more relevant in military, governmental, and commercial contexts. As a consequence, there is an increasing need for decision making support tools for real time air traffic management in complex environments where several conflicts might happen at the same time.

According to the International Civil Aviation Organization [1], aircraft must be separated by at least 5 NM horizontally and 1000 ft vertically during the flight. A pair of aircraft violating at least one of these rules are said to be in conflict. In this work, we consider the problem of aircraft separation in the planar space since altitude change maneuvers are usually neglected in short-time planning (as they are expensive and uncomfortable for passengers).

We propose a novel exact algorithm, based on mathematical programming techniques, that adjust the heading angles and nominal speeds to provide conflict-free optimal trajectories. When only heading angle or speed changes are allowed, the problem can be written as a mixed integer program [3]. However, when both maneuvers are considered, the formulation becomes nonlinear. Different simplifications, such as maneuver discretization [2], separated phases for heading and speed changes [5] or problem relaxations [4], has been considered to approach the problem. We propose to use a tailored branch and cut that solves the deconfliction problem under relaxed (and polyhedral) feasible regions for the vectors of velocities.

2 Problem statement

Let $\mathcal{A}$ be a set of aircraft flying at the same altitude in a given time window $[0, T]$ and air sector. We will denote by D the safety distance that must be maintained between a pair of

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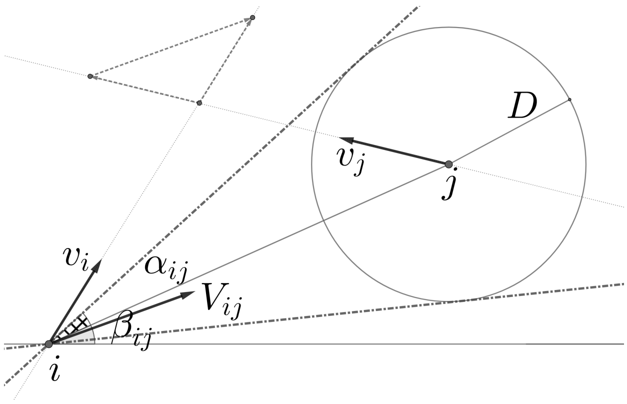


FIG. 1: Geometric analysis of conflict between two aircraft based on relative velocity

aircraft. For every $i \in \mathcal{A}$, $v_i = (v_{i,x}, v_{i,y})$ is the vector of velocity of i and, $V_{ij} := v_i - v_j$ is the vector of velocity of i relative to aircraft j, for each $j \in \mathcal{A}$, $i < j$. We denote by v the vector of all the velocities.

Figure 1 shows a geometric analysis of the conflict between two aircraft $i, j \in \mathcal{A}$ based on the relative velocity vector V_{ij}. When V_{ij} points somewhere inside the disk of radius D around j, there is a conflict between the two aircraft. To avoid the conflict, the velocities of i and j must be modified so that the new relative vector, V_{ij}, lies outside the disk. The two tangents between i and the disk (dashed-dot lines in Figure 1) delimit the slope of such a suitable vector V_{ij}. Based on this analysis and using trigonometry, it can be seen that the separation condition for i and j can be written as in [3]

$$\frac{V_{ij,y}}{V_{ij,x}} \geq \tan(\beta_{ij} + \alpha_{ij}) \qquad \text{or} \qquad \frac{V_{ij,y}}{V_{ij,x}} \leq \tan(\beta_{ij} - \alpha_{ij}), \tag{1}$$

where $V_{ij} = (V_{ij,x}, V_{ij,y})$, β_{ij} is the angle between the segment ij and the x-axis and $\alpha_{ij} := \arcsin\left(\frac{D}{d(i,j)}\right)$, with $d(\cdot,\cdot)$ denoting the Euclidean distance. This disjunctive constraint is usually modeled with binary variables, while we present a different approach.

Other than conflict resolution itself, another important feature that one needs to model are the bounds for the maneuvers. Due to safety reasons, both speed and heading angles changes have to be kept under some specified limits. If we call $\underline{v}_i, \bar{v}_i$ and $\underline{\theta}_i, \bar{\theta}_i$ the lower and upper bounds for the modified speed and heading angle respectively, the bounding constraints are:

$$\underline{v}_i^2 \leq v_{i,x}^2 + v_{i,y}^2 \leq \bar{v}_i^2 \quad \forall i \in \mathcal{A} \tag{2}$$

$$\tan \underline{\theta}_i \leq \frac{v_{i,y}}{v_{i,x}} \leq \tan \bar{\theta}_i \quad \forall i \in \mathcal{A}, \tag{3}$$

where we assume the angles in the interval $[-\pi/2, \pi/2]$.

Regarding the objective function, there is a wide range of aspects to consider such as the deviations from nominal trajectories, fuel consumption, delay reduction, or fairness. The algorithm we describe is conceptually valid for any objective, so we will refer to this function as $f(v)$. Of course, the algorithm performance will depend on the mathematical representation of f.

3 Branch and cut algorithm

We propose to use a tailored branch and cut method that solves different relaxations of the problem. At the root node, we have the following problem:

$$(P_0) = \begin{cases} \min & f(v) \\ \text{s.t.} & (v_{i,x}, v_{i,y}) \in F_i^0 \end{cases}$$

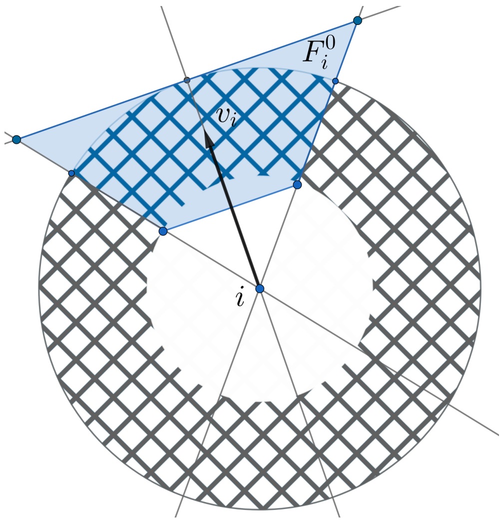


FIG. 2: Relaxation of the feasible region for v_i

The only set of constraints in (P_0) are bounding constraints. More precisely, for each aircraft $i \in \mathcal{A}$, F_i^0 is a polyhedron enclosing the feasible region for the vector v_i, i.e., F_i^0 is a relaxation of (2) together with (3). This is illustrated on Figure 2. Indeed, the grey squared region is an annulus that corresponds to constraints (2) on the magnitude of v_i. The blue sector of the annulus represents the admissible heading angles and corresponds to constraints (3). Note that the feasible region, the blue sector of the anunulus, is nonconvex. Polyhedron F_i^0, shown also in blue, is constructed by taking the segment between the heads of $(\underline{v}_i \cos \underline{\theta}_i, \underline{v}_i \sin \underline{\theta}_i)$ and $(\underline{v}_i \cos \bar{\theta}_i, \underline{v}_i \sin \bar{\theta}_i)$ and the tangent to the outer disk that is perpendicular to the bisector of these two vectors.

We denote by $\tilde{v}$ the optimal solution of the problem at the current node of the branching tree, starting by (P_0) at the root. The list of open subproblems is denoted by Π, initially $\Pi := \{(\mathrm{P}_0)\}$. We set the current best objective value (upper bound of f) to infinity, $f^* = +\infty$. The proposed algorithm is as follows:

While Π is not empty:

1. (P)$\leftarrow$ extract a problem from Π.
2. Solve (P), i.e., find its optimal solution $\tilde{v}$. If $f(\tilde{v}) \geq f^*$, discard the node (go to step 1).
3. **Repeat until** $\tilde{v}_{i,x}^2 + \tilde{v}_{i,y}^2 \leq \bar{v}_i^2 \quad \forall i \in \mathcal{A}$

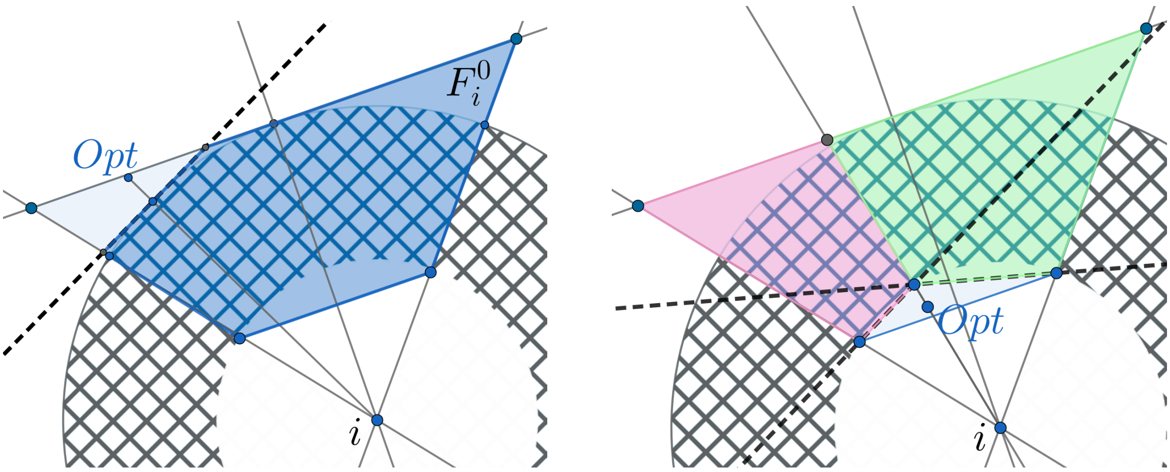
   3a. **Cut**: **For all** $i \in \mathcal{A}$ such that $\tilde{v}_{i,x}^2 + \tilde{v}_{i,y}^2 \geq \bar{v}_i^2$, add a cut to (P). The cut, illustrated on Figure 3a with a dashed line, is the tangent to the outer disk around i at the point which is in the same line as i and the head of $\tilde{v}_i$.

   3b. Find the optimal solution of (P), $\tilde{v}$. If $f(\tilde{v}) \geq f^*$, discard the node and go to step 1.

4. **Branch**:

   4a) If, for some $i \in \mathcal{A}$, $\tilde{v}_{i,x}^2 + \tilde{v}_{i,y}^2 \leq \underline{v}_i^2$, add disjunctive cuts to exclude this solution. In this case we generate two subproblems, (P') and (P"), with feasible regions depicted in pink and green in Figure 3b (the disjunctive cuts are illustrated with dashed lines). We add (P') and (P") to Π.

   4b) Otherwise, if, for some pair $i, j \in \mathcal{A}$, condition (1) is not satisfied, we generate two subproblems, (P') and (P"), each of them including one of the inequalities in (1), and we add them to Π.



(a) Magnitude above the upper bound

(b) Magnitude below the upper bound

FIG. 3: Cuts to eliminate unfeasible solutions

4c. Otherwise, if $f(\tilde{v}) < f^*$, update the incumbent value $f^* := f(\tilde{v})$.

End-While

4 Conclusion and perspectives

The presented algorithm defines an exact procedure for aircraft deconfliction were (simultaneous) speed and heading angle changes are allowed. It consists of a branch and cut where a linearly constrained subproblem with continuous variables is solved at every node. As future perspectives, we contemplate improvements of the algorithm such as the use of valid inequalities (cuts) or the preprocessing of a set $\mathcal{C} \subseteq \mathcal{A} \times \mathcal{A}$ of potential conflicting pairs. Another interesting direction to explore is the performance of the algorithm under different types of objectives.

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