On superperfection of edge intersection graphs of paths

Hervé Kerivin and **Annegret K. Wagler**

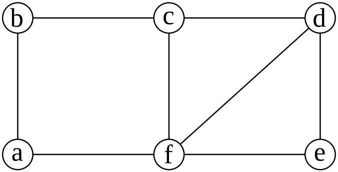
Université Clermont Auvergne
LIMOS, UMR 6158 CNRS
Clermont-Ferrand, France

Cologne Twente Workshop 2020
Sept. 14-16, 2020

This work was supported by the French National Research Agency grant ANR-17-CE25-0006, project FLEXOPTIM.

The routing and spectrum assignment problem

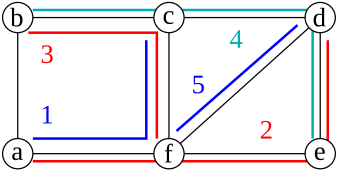
Given an optical network G, an optical spectrum $S = \{1, \ldots, \bar{s}\}$, and a set $\mathcal{D}$ of demands $k = (o_k, d_k, w_k)$.



k	$o_k \longrightarrow d_k$	w_k
1	$a \longrightarrow c$	2
2	$a \longrightarrow d$	1
3	$b \longrightarrow f$	2
4	$b \longrightarrow e$	1
5	$d \longrightarrow f$	3

The routing and spectrum assignment problem

Given an optical network G, an optical spectrum $S = \{1, \ldots, \bar{s}\}$, and a set $\mathcal{D}$ of demands $k = (o_k, d_k, w_k)$.



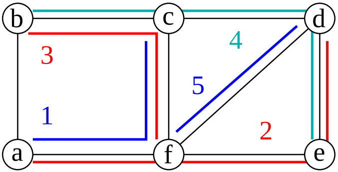
k	$o_k \longrightarrow d_k$	w_k
1	$a \longrightarrow c$	2
2	$a \longrightarrow d$	1
3	$b \longrightarrow f$	2
4	$b \longrightarrow e$	1
5	$d \longrightarrow f$	3

Find for each demand $k = (o_k, d_k, w_k)$

- an (o_k,d_k)-path P_k in G **(routing)**,

The routing and spectrum assignment problem

Given an optical network G, an optical spectrum $S = \{1, \ldots, \bar{s}\}$, and a set $\mathcal{D}$ of demands $k = (o_k, d_k, w_k)$.



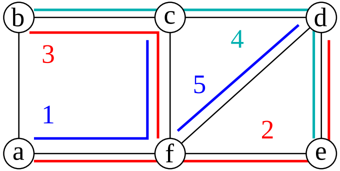
k	$o_k \rightarrow d_k$	w_k	S_k
1	$a \rightarrow c$	2	1 2
2	$a \rightarrow d$	1	3
3	$b \rightarrow f$	2	3 4
4	$b \rightarrow e$	1	1
5	$d \rightarrow f$	3	1 2 3

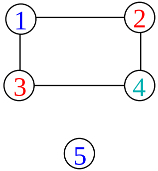
Find for each demand $k = (o_k, d_k, w_k)$

- an (o_k,d_k)-path P_k in G **(routing)**,
- an interval $S_k \subseteq S$ of w_k consecutive frequency slots s.t. $S_k \cap S_{k'} = \emptyset$ for each demand k' routed along an edge of P_k **(spectrum assignment)**.

Interval colorings of edge intersection graphs of paths

For each routing $\mathcal{P} = \{P_k : k \in \mathcal{D}\}$, the spectrum assignment corresponds to an **interval coloring** of the **edge intersection graph** $I(\mathcal{P})$:





k	w_k	S_k
1	2	1 2
2	1	3
3	2	3 4
4	1	1
5	3	1 2 3

- Each path $P_k \in \mathcal{P}$ becomes a node k of $I(\mathcal{P})$, two nodes k and k' are joined by an edge if the corresponding paths P_k and $P_{k'}$ share an edge in G.
- Each node k of $I(\mathcal{P})$ receives a frequency interval $S_k \subseteq S$ of w_k consecutive frequency slots s.t. $S_k \cap S_{k'} = \emptyset$ when k and k' are adjacent.

Superperfect graphs

Definition

- **interval chromatic number** $\chi_I(G, \mathbf{w})$: minimum spectrum width s.t. G weighted with $\mathbf{w}$ has a proper interval coloring
- **weighted clique number** $\omega(G, \mathbf{w})$: maximum weight of a clique in G weighted with $\mathbf{w}$

We have $\omega(G, \mathbf{w}) \leq \chi_I(G, \mathbf{w})$ for all graphs G and weight vectors $\mathbf{w} \in \mathbb{N}_0^{|G|}$.

Definition

A graph G is **superperfect** if $\omega(G, \mathbf{w}) = \chi_I(G, \mathbf{w})$ for all weight vectors $\mathbf{w} \in \mathbb{N}_0^{|G|}$.

Remark.

- Every superperfect graph is perfect (for the latter, we have the equality $\omega(G, \mathbf{w}) = \chi_I(G, \mathbf{w})$ for all weight vectors $\mathbf{w} \in \{0, 1\}^{|G|}$).
- Every comparability graph is superperfect (Hoffman 1974).

Superperfect graphs

Definition

- **interval chromatic number** $\chi_I(G, \mathbf{w})$: minimum spectrum width s.t. G weighted with $\mathbf{w}$ has a proper interval coloring
- **weighted clique number** $\omega(G, \mathbf{w})$: maximum weight of a clique in G weighted with $\mathbf{w}$

We have $\omega(G, \mathbf{w}) \leq \chi_I(G, \mathbf{w})$ for all graphs G and weight vectors $\mathbf{w} \in \mathbb{N}_0^{|G|}$.

Definition

A graph G is **superperfect** if $\omega(G, \mathbf{w}) = \chi_I(G, \mathbf{w})$ for all weight vectors $\mathbf{w} \in \mathbb{N}_0^{|G|}$.

Remark.

- Every superperfect graph is perfect (for the latter, we have the equality $\omega(G, \mathbf{w}) = \chi_I(G, \mathbf{w})$ for all weight vectors $\mathbf{w} \in \{0, 1\}^{|G|}$).
- Every comparability graph is superperfect (Hoffman 1974).

Superperfect graphs

Definition

- **interval chromatic number** $\chi_I(G, \mathbf{w})$: minimum spectrum width s.t. G weighted with $\mathbf{w}$ has a proper interval coloring
- **weighted clique number** $\omega(G, \mathbf{w})$: maximum weight of a clique in G weighted with $\mathbf{w}$

We have $\omega(G, \mathbf{w}) \leq \chi_I(G, \mathbf{w})$ for all graphs G and weight vectors $\mathbf{w} \in \mathbb{N}_0^{|G|}$.

Definition

A graph G is **superperfect** if $\omega(G, \mathbf{w}) = \chi_I(G, \mathbf{w})$ for all weight vectors $\mathbf{w} \in \mathbb{N}_0^{|G|}$.

Remark.

- Every superperfect graph is perfect (for the latter, we have the equality $\omega(G, \mathbf{w}) = \chi_I(G, \mathbf{w})$ for all weight vectors $\mathbf{w} \in \{0, 1\}^{|G|}$).
- Every comparability graph is superperfect (Hoffman 1974).

Non-comparability graphs

Every non-superperfect graph contains a non-comparability graph.

Theorem (Gallai 1967)

The minimal non-comparability graphs are as follows:

- odd holes C_{2k+1} for $k \geq 2$ and antiholes $\overline{C}_n$ for $n \geq 6$,

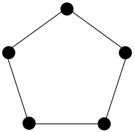
Non-comparability graphs

Every non-superperfect graph contains a non-comparability graph.

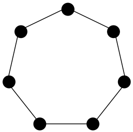
Theorem (Gallai 1967)

The minimal non-comparability graphs are as follows:

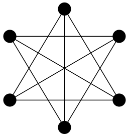
- odd holes C_{2k+1} for $k \geq 2$ and antiholes $\overline{C}_n$ for $n \geq 6$,



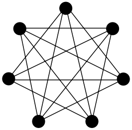
$C_5 = \bar{C}_5$



C_7



$\bar{C}_6$



$\bar{C}_7$

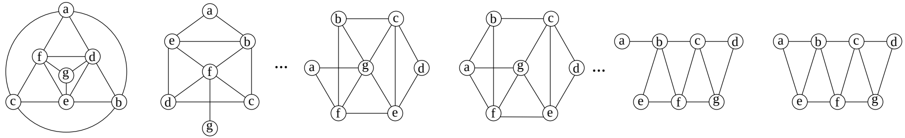
Non-comparability graphs

Every non-superperfect graph contains a non-comparability graph.

Theorem (Gallai 1967)

The minimal non-comparability graphs are as follows:

- odd holes C_{2k+1} for $k \geq 2$ and antiholes $\overline{C}_n$ for $n \geq 6$,
- ten 7-node graphs $\overline{A}_1, \ldots, \overline{A}_{10}$,



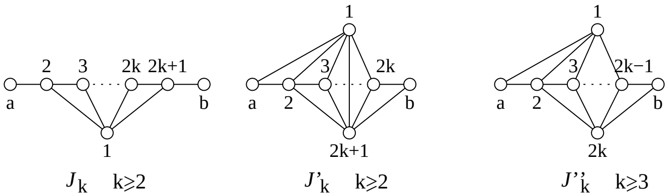
Non-comparability graphs

Every non-superperfect graph contains a non-comparability graph.

Theorem (Gallai 1967)

The minimal non-comparability graphs are as follows:

- odd holes C_{2k+1} for $k \geq 2$ and antiholes $\overline{C}_n$ for $n \geq 6$,
- ten 7-node graphs $\overline{A}_1, \ldots, \overline{A}_{10}$,
- the graphs J_k, J'_k for $k \geq 2$, and J''_k for $k \geq 3$,



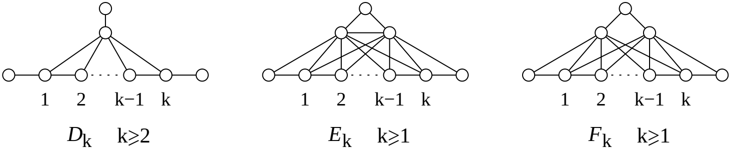
Non-comparability graphs

Every non-superperfect graph contains a non-comparability graph.

Theorem (Gallai 1967)

The minimal non-comparability graphs are as follows:

- odd holes C_{2k+1} for $k \geq 2$ and antiholes $\overline{C}_n$ for $n \geq 6$,
- ten 7-node graphs $\overline{A}_1, \ldots, \overline{A}_{10}$,
- the graphs J_k, J'_k for $k \geq 2$, and J''_k for $k \geq 3$,
- the complements of D_k for $k \geq 2$ and of E_k, F_k for $k \geq 1$.



Non-superperfect graphs

We call a minimal non-superperfect graph **basic** if it is minimal non-comparability.

Theorem (Golumbic 1980, Andreae 1985)

The basic minimal non-superperfect graphs are as follows:

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,

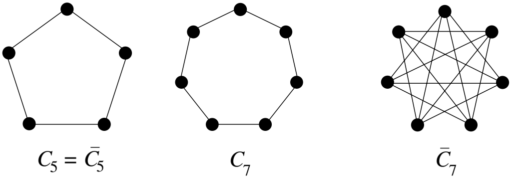
Non-superperfect graphs

We call a minimal non-superperfect graph **basic** if it is minimal non-comparability.

Theorem (Golumbic 1980, Andreae 1985)

The basic minimal non-superperfect graphs are as follows:

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,



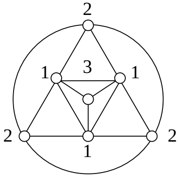
Non-superperfect graphs

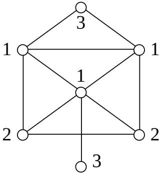
We call a minimal non-superperfect graph **basic** if it is minimal non-comparability.

Theorem (Golumbic 1980, Andreae 1985)

The basic minimal non-superperfect graphs are as follows:

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ and $\overline{A}_2$,





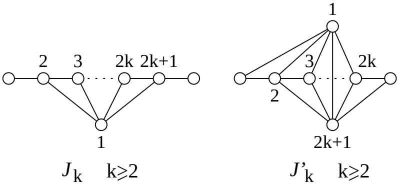
Non-superperfect graphs

We call a minimal non-superperfect graph **basic** if it is minimal non-comparability.

Theorem (Golumbic 1980, Andreae 1985)

The basic minimal non-superperfect graphs are as follows:

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ and $\overline{A}_2$,
- the graphs J_k for $k \geq 2$ and J'_k for $k \geq 3$,



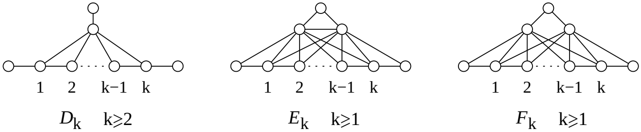
Non-superperfect graphs

We call a minimal non-superperfect graph **basic** if it is minimal non-comparability.

Theorem (Golumbic 1980, Andreae 1985)

The basic minimal non-superperfect graphs are as follows:

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ and $\overline{A}_2$,
- the graphs J_k for $k \geq 2$ and J'_k for $k \geq 3$,
- the complements of D_k for $k \geq 2$ and of E_k, F_k for $k \geq 1$.



The complete list of all minimal non-superperfect graphs is not yet known.

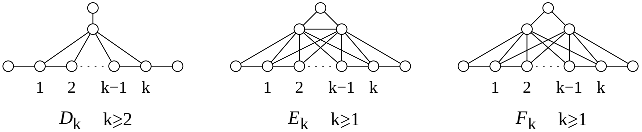
Non-superperfect graphs

We call a minimal non-superperfect graph **basic** if it is minimal non-comparability.

Theorem (Golumbic 1980, Andreae 1985)

The basic minimal non-superperfect graphs are as follows:

- odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ and $\overline{A}_2$,
- the graphs J_k for $k \geq 2$ and J'_k for $k \geq 3$,
- the complements of D_k for $k \geq 2$ and of E_k, F_k for $k \geq 1$.



The complete list of all minimal non-superperfect graphs is not yet known.

Superperfectiou and spectrum width

The clique bound

$$\omega(G,\mathcal{D}) = \min\{\omega(I(\mathcal{P}),\mathbf{w}) : \mathcal{P} \text{ possible routing of demands } \mathcal{D} \text{ in } G\}$$

is a lower bound of the minimum spectrum width

$$\chi_I(G,\mathcal{D}) = \min\{\chi_I(I(\mathcal{P}),\mathbf{w}) : \mathcal{P} \text{ possible routing of demands } \mathcal{D} \text{ in } G\}$$

of any solution of the RSA problem.

Goal

Examine, for different networks G, whether there exists a solution of the RSA problem with

$$\omega(G,\mathcal{D}) = \chi_I(G,\mathcal{D})$$

which depends on the occurrence of non-superperfect graphs in $I(\mathcal{P})$.

Superperfectiou and spectrum width

The clique bound

$$\omega(G, \mathcal{D}) = \min\{\omega(I(\mathcal{P}), \mathbf{w}) : \mathcal{P} \text{ possible routing of demands } \mathcal{D} \text{ in } G\}$$

is a lower bound of the minimum spectrum width

$$\chi_I(G, \mathcal{D}) = \min\{\chi_I(I(\mathcal{P}), \mathbf{w}) : \mathcal{P} \text{ possible routing of demands } \mathcal{D} \text{ in } G\}$$

of any solution of the RSA problem.

Goal

Examine, for different networks G, whether there exists a solution of the RSA problem with

$$\omega(G, \mathcal{D}) = \chi_I(G, \mathcal{D})$$

which depends on the occurrence of non-superperfect graphs in $I(\mathcal{P})$.

A hierarchy of considered networks

For some networks G, the edge intersection graphs form well-studied graph classes:

all graphs $= I(\mathcal{P})$ with $\mathcal{P}$ in **grids**[3)]

$\Uparrow$

$I(\mathcal{P})$ with $\mathcal{P}$ in **optical networks**[1)]

$\Uparrow$ $\Uparrow$

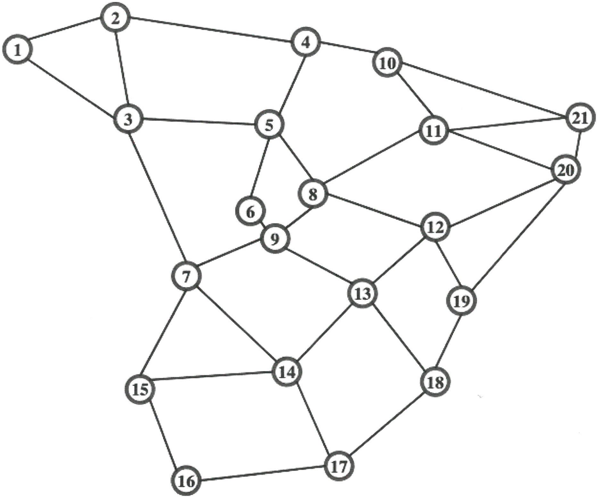
EPT graphs $= I(\mathcal{P})$ with $\mathcal{P}$ in **trees**[2)] **CA graphs** $= I(\mathcal{P})$ with $\mathcal{P}$ in **cycles**

$\Uparrow$ $\Uparrow$

interval graphs $= I(\mathcal{P})$ with $\mathcal{P}$ in **paths**

1) optical networks = sparse planar graphs with small maximum degree
2) EPT graphs introduced and studied by Golumbic & Jamison 1985
3) follows from a result of Golumbic, Lipshteyn & Stern 2009

An optical network



The Spanish Telefónica network

Outline

Outline

Superperfection of interval graphs

If $\mathcal{P}$ is a set of paths in a **path**, then $I(\mathcal{P})$ is an **interval graph**.

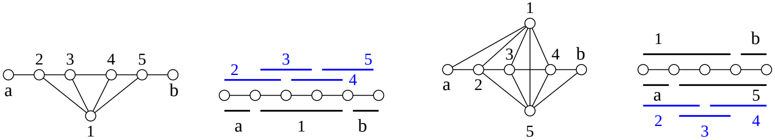
Theorem (Kerivin & Wagler 2020)

An interval graph can contain the following basic non-superperfect graphs:

- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,

Proof (idea):

- give according path representations for the affirmative cases



Superperfection of interval graphs

If $\mathcal{P}$ is a set of paths in a **path**, then $I(\mathcal{P})$ is an **interval graph**.

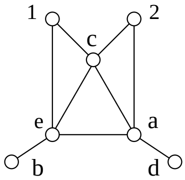
Theorem (Kerivin & Wagler 2020)

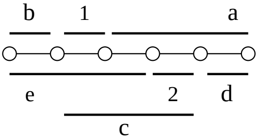
An interval graph can contain the following basic non-superperfect graphs:

- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- $\overline{E}_2$,

Proof (idea):

- give according path representations for the affirmative cases





Superperfection of interval graphs

If $\mathcal{P}$ is a set of paths in a **path**, then $I(\mathcal{P})$ is an **interval graph**.

Theorem (Kerivin & Wagler 2020)

An interval graph can contain the following basic non-superperfect graphs:

- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- $\overline{E}_2$,

but none of the other minimal non-comparability non-superperfect graphs.

Proof (idea):

- give according path representations for the affirmative cases
- use the complete list of minimal non-interval graphs by Lekkerkerker and Boland (1962) to exclude all others:
  - odd holes, $\overline{D}_2$, $\overline{E}_1$ are minimal non-interval graphs
  - all other minimal non-comparability superperfect graphs contain C_4

Non-basic non-superperfect interval graphs

Any non-basic non-superperfect graph has to contain a minimal non-comparability superperfect graph as proper induced subgraph.

We observe that any non-basic minimal non-superperfect interval graph can contain

- no even antihole $\overline{C}_{2k}$ for $k \geq 3$ (as they all contain a C_4),
- none of the graphs J''_k for all $k \geq 3$ (as they all contain a C_4),
- none of the graphs $\overline{A}_3, \ldots \overline{A}_8$ (as they all contain a C_4),

but only $\overline{A}_9$, $\overline{A}_{10}$ and J'_2.

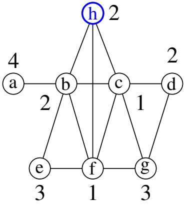
Non-basic non-superperfect interval graphs

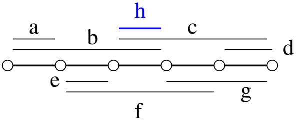
Any non-basic non-superperfect graph has to contain a minimal non-comparability superperfect graph as proper induced subgraph.

We observe that any non-basic minimal non-superperfect interval graph can contain

- no even antihole $\overline{C}_{2k}$ for $k \geq 3$ (as they all contain a C_4),
- none of the graphs J''_k for all $k \geq 3$ (as they all contain a C_4),
- none of the graphs $\overline{A}_3, \ldots \overline{A}_8$ (as they all contain a C_4),

but only $\overline{A}_9$, $\overline{A}_{10}$ and J'_2.





Outline

Superperfection of EPT graphs

Theorem (Kerivin & Wagler 2020)

An EPT graph can contain the following basic non-superperfect graphs:

- odd holes C_{2k+1} for $k \geq 2$, but no odd antiholes $\overline{C}_{2k+1}$ for $k \geq 3$,
- $\overline{A}_1$ and $\overline{A}_2$,
- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- $\overline{D}_2$, $\overline{D}_3$, $\overline{E}_1$, $\overline{E}_2$, $\overline{E}_3$, $\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$, but none of $\overline{D}_k$, $\overline{E}_k$, $\overline{F}_k$ for $k \geq 4$.

Proof (idea):

- the assertion for C_{2k+1} and $\overline{C}_{2k+1}$ follows from Golumbic & Jamison 1985,
- give according path representations for all other affirmative cases,
- exhibit a $\overline{P}_6$ (which is not an EPT graph by Golumbic & Jamison 1985) as common subgraph of the remaining cases.

Theorem (Kerivin & Wagler 2020)

An EPT graph can contain the following basic non-superperfect graphs:

- odd holes C_{2k+1} for $k \geq 2$, but no odd antiholes $\overline{C}_{2k+1}$ for $k \geq 3$,
- $\overline{A}_1$ and $\overline{A}_2$,
- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- $\overline{D}_2$, $\overline{D}_3$, $\overline{E}_1$, $\overline{E}_2$, $\overline{E}_3$, $\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$, but none of $\overline{D}_k$, $\overline{E}_k$, $\overline{F}_k$ for $k \geq 4$.

Proof (idea):

- the assertion for C_{2k+1} and $\overline{C}_{2k+1}$ follows from Golumbic & Jamison 1985,
- give according path representations for all other affirmative cases,
- exhibit a $\overline{P}_6$ (which is not an EPT graph by Golumbic & Jamison 1985) as common subgraph of the remaining cases.

Non-basic non-superperfect EPT graphs

Any non-basic non-superperfect graph has to contain a minimal non-comparability superperfect graph as proper induced subgraph.

We observe that any non-basic minimal non-superperfect EPT graph can contain J_2', $\overline{C}_6$ and the graphs $\overline{A}_3, \ldots \overline{A}_6$, $\overline{A}_8, \ldots \overline{A}_{10}$ but

- no even antihole $\overline{C}_{2k}$ for $k \geq 4$,
- none of the graphs J_k'' for all $k \geq 3$,
- not $\overline{A}_7$,

as they all contain a non-EPT graph found by Golumbic & Jamison 1985.

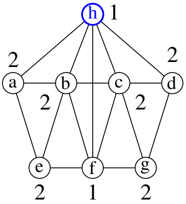
Non-basic non-superperfect EPT graphs

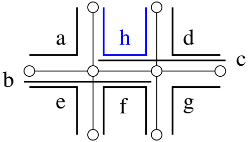
Any non-basic non-superperfect graph has to contain a minimal non-comparability superperfect graph as proper induced subgraph.

We observe that any non-basic minimal non-superperfect EPT graph can contain J_2', $\overline{C}_6$ and the graphs $\overline{A}_3, \ldots \overline{A}_6$, $\overline{A}_8, \ldots \overline{A}_{10}$ but

- no even antihole $\overline{C}_{2k}$ for $k \geq 4$,
- none of the graphs J_k'' for all $k \geq 3$,
- not $\overline{A}_7$,

as they all contain a non-EPT graph found by Golumbic & Jamison 1985.





Outline

Superperfection of CA graphs

Theorem (Kerivin & Wagler 2019)

A CA graph can contain the following basic non-superperfect graphs:

- all odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ but not $\overline{A}_2$,
- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- $\overline{D}_2$, $\overline{D}_3$, $\overline{D}_4$, but not the graphs $\overline{D}_k$ for $k \geq 5$,
- $\overline{E}_1$ and $\overline{E}_2$, but not the graphs $\overline{E}_k$ for $k \geq 3$,
- $\overline{F}_2$, but not $\overline{F}_1$ neither the graphs $\overline{F}_k$ for $k \geq 3$.

Proof (idea):

- the assertion for C_{2k+1} and $\overline{C}_{2k+1}$ is well-known
- give according path representations for all affirmative cases,
- exhibit a minimal non-CA graph as induced subgraph of the remaining cases.

N.B.: $\overline{E}_3$ is a new minimal non-circular-arc graph.

Superperfection of CA graphs

Theorem (Kerivin & Wagler 2019)

A CA graph can contain the following basic non-superperfect graphs:

- all odd holes C_{2k+1} and odd antiholes $\overline{C}_{2k+1}$ for $k \geq 2$,
- $\overline{A}_1$ but not $\overline{A}_2$,
- the graphs J_k for all $k \geq 2$ and J'_k for all $k \geq 3$,
- $\overline{D}_2$, $\overline{D}_3$, $\overline{D}_4$, but not the graphs $\overline{D}_k$ for $k \geq 5$,
- $\overline{E}_1$ and $\overline{E}_2$, but not the graphs $\overline{E}_k$ for $k \geq 3$,
- $\overline{F}_2$, but not $\overline{F}_1$ neither the graphs $\overline{F}_k$ for $k \geq 3$.

Proof (idea):

- the assertion for C_{2k+1} and $\overline{C}_{2k+1}$ is well-known
- give according path representations for all affirmative cases,
- exhibit a minimal non-CA graph as induced subgraph of the remaining cases.

N.B.: $\overline{E}_3$ is a new minimal non-circular-arc graph.

Non-basic non-superperfect CA graphs

Any non-basic non-superperfect graph has to contain a minimal non-comparability superperfect graph as proper induced subgraph.

We observe that any non-basic minimal non-superperfect CA graph can contain J_2' all of the graphs $\overline{A}_3, \ldots \overline{A}_{10}$ but

- no even antihole $\overline{C}_{2k}$ for $k \geq 3$,
- none of the graphs J_k'' for all $k \geq 3$,

as they all contain a non-CA graph.

N.B.: J_3'' is a new minimal non-circular-arc graph.

Non-basic non-superperfect CA graphs

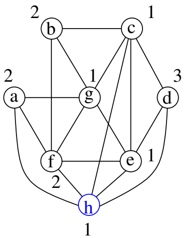
Any non-basic non-superperfect graph has to contain a minimal non-comparability superperfect graph as proper induced subgraph.

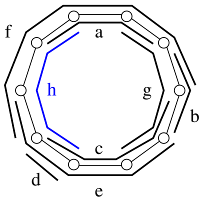
We observe that any non-basic minimal non-superperfect CA graph can contain J_2' all of the graphs $\overline{A}_3, \ldots \overline{A}_{10}$ but

- no even antihole $\overline{C}_{2k}$ for $k \geq 3$,
- none of the graphs J_k'' for all $k \geq 3$,

as they all contain a non-CA graph.

N.B.: J_3'' is a new minimal non-circular-arc graph.





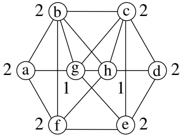
Outline

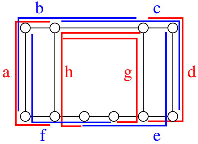
Superperfection in the case of optical networks

Theorem (Kerivin & Wagler 2019)

All basic non-superperfect graphs can occur in edge intersection graphs $I(\mathcal{P})$ of sets $\mathcal{P}$ of paths in optical networks G.

- We found many of them in $I(\mathcal{P})$'s of the Spanish Telefónica network
- There are more non-basic non-superperfect graphs, e.g.:





- We expect that **all** minimal non-superperfect graphs can occur in $I(\mathcal{P})$ of paths in networks, as soon as the networks G satisfy minimal survivability conditions concerning edge or node failures.

Outline

Concluding remarks

Our study of combinatorial properties behind the RSA problem showed that

- there are problem instances with $\omega(G, \mathcal{D}) < \chi_I(G, \mathcal{D})$,
- it is often hard to certify optimality of a solution.

Our lines of future research include to

- further study the representation of cliques in $I(\mathcal{P})$ with $\mathcal{P}$ in optical networks,
- find more non-basic non-superperfect graphs,
- estimate the gap between $\omega(I(\mathcal{P}), \mathbf{w})$ and $\chi_I(I(\mathcal{P}), \mathbf{w})$ in terms of χ_I-binding functions f with

$$\chi_I(I(\mathcal{P}), \mathbf{w}) \leq f(\omega(I(\mathcal{P}), \mathbf{w}))$$

  for edge intersection graphs $I(\mathcal{P})$ in a certain class of networks and all possible non-negative integral weights $\mathbf{w}$,
- turn this knowledge into optimality certificates.

Concluding remarks

Our study of combinatorial properties behind the RSA problem showed that

- there are problem instances with $\omega(G, \mathcal{D}) < \chi_I(G, \mathcal{D})$,
- it is often hard to certify optimality of a solution.

Our lines of future research include to

- further study the representation of cliques in $I(\mathcal{P})$ with $\mathcal{P}$ in optical networks,
- find more non-basic non-superperfect graphs,
- estimate the gap between $\omega(I(\mathcal{P}), \mathbf{w})$ and $\chi_I(I(\mathcal{P}), \mathbf{w})$ in terms of χ_I-binding functions f with
$$\chi_I(I(\mathcal{P}), \mathbf{w}) \leq f(\omega(I(\mathcal{P}), \mathbf{w}))$$
for edge intersection graphs $I(\mathcal{P})$ in a certain class of networks and all possible non-negative integral weights $\mathbf{w}$,
- turn this knowledge into optimality certificates.