Edge tree spanners

Fernanda Couto – Universidade Federal Rural do Rio de Janeiro, Brasil
Luís Cunha – Universidade Federal Fluminense, Brasil
Daniel Posner – Universidade Federal Rural do Rio de Janeiro, Brasil

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A min-max problem

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- However, it can become a challenge if we add some constraints on it
- In this work, we deal with a problem that makes two opposite optimality analysis
  - Goal: determining trees that minimize the maximum distance between vertices (or edges) of connected graphs.

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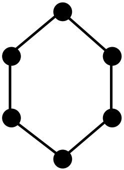
The MSST problem
Instance: A connected graph G and an integer t.
Question: Is G $\sigma_T(G)$-admissible graph, $\sigma_T(G) \leq t$?

Examples



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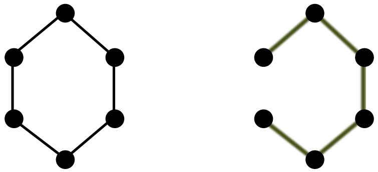

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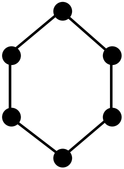

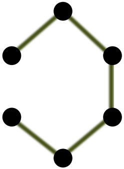

$$\sigma_T(C_6) = 5$$

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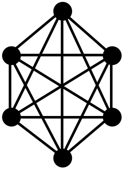

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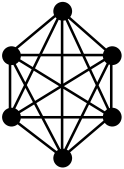

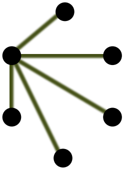

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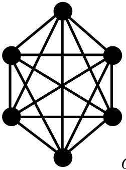
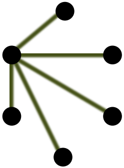


$\sigma_T(K_6) = 2$



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Equivalently:
Determining the edge stretch index of G is equivalent to determine the stretch index of $L(G)$.

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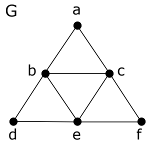

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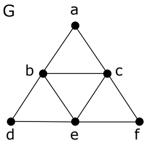

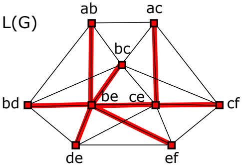

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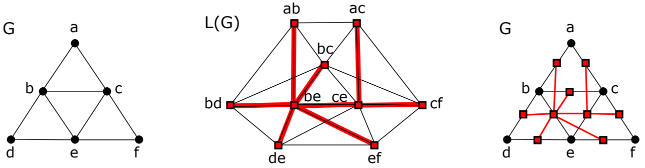

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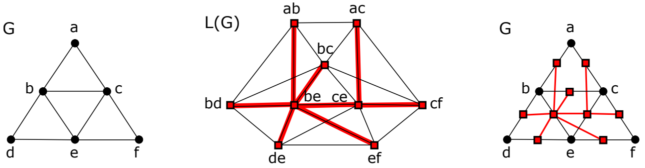

G is edge 3-admissible.

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- Edge tree-spanners:

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- prove that Edge 8-admissibility problem is NP-complete, even for bipartite graphs.
  - Hence, we can adapt such a NP-completeness in order to prove that Edge $2k$-admissibility problem is NP-complete, for $k \geq 5$.

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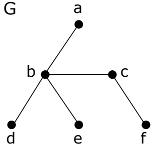

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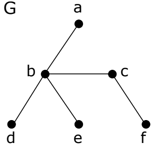

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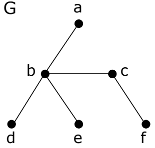

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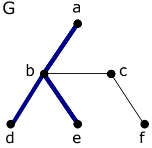

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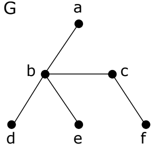

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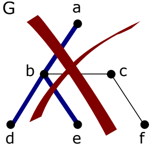

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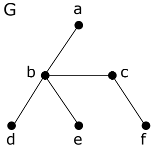

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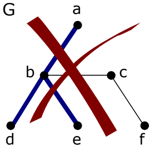

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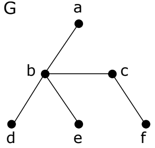

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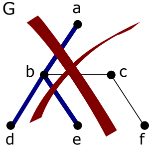

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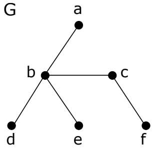

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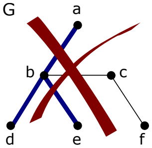

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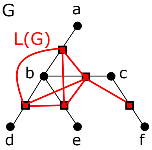

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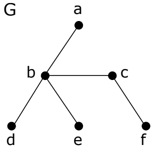

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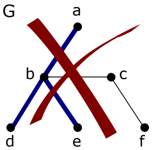

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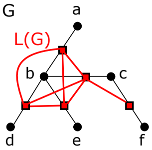

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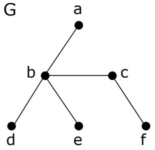

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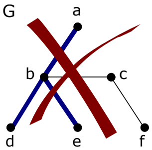

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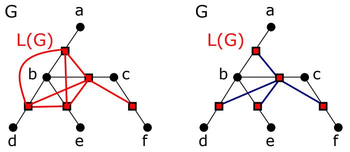

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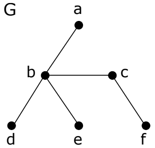

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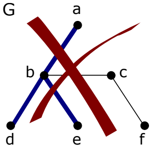

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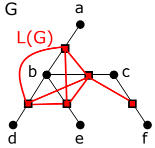

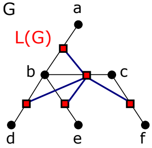

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Corollary: Edge 2-admissibility is a polynomial time solvable problem.

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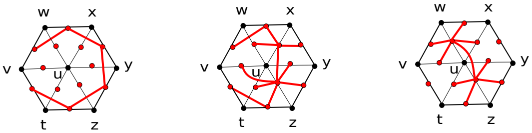

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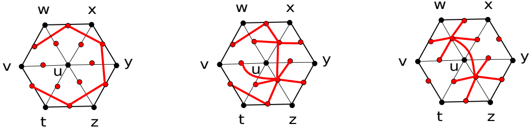

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The addition of u does not imply in a edge 3-admissible graph.

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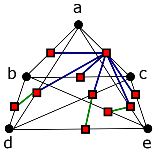

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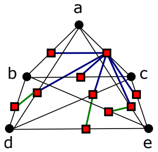

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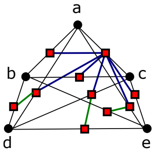

2. K_n are edge 4-admissible, for $n \geq 5$.

3. Edge 3-admissibility is hereditary and any K_n, for $n \geq 5$, contains a K_5 as induced subgraph, which is not edge 3-admissible, hence, K_n is not edge 3-admissible, as well.

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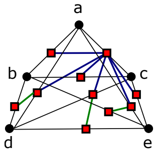

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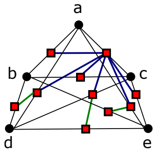

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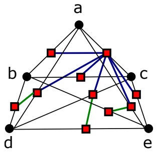

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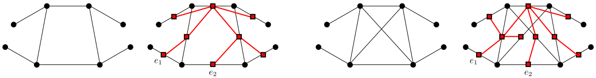

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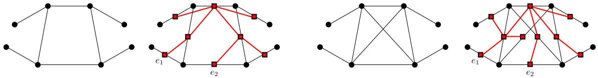

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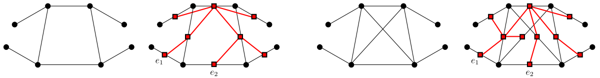

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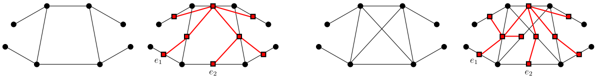

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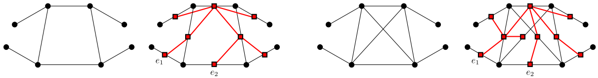

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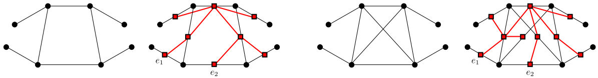

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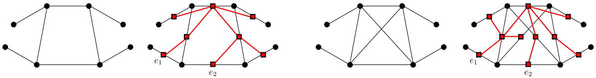

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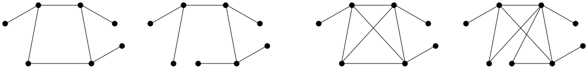

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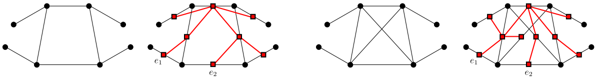

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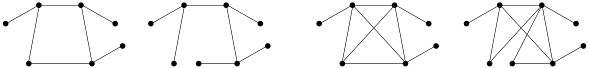

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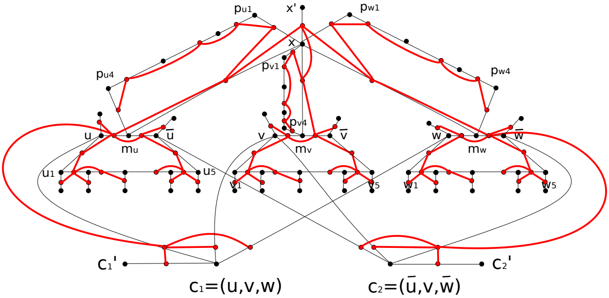

Graph obtained from instance $I = (\{u, v, w\}, \{(u, v, w), (\overline{u}, v, \overline{w}\})$ and an edge tree 8-spanner of it in red.

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Thank you!