A Lagrangian approach to Chance Constrained Routing with Local Broadcast

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Introduction

Mobile cellular networks play a pivotal role in emerging Internet of Thing (IoT) applications.

Many of these applications are characterized by the need of **routing messages** :

- within a given local area
- constraints about timeliness
- constraints about reliability (i.e., probability of reception)
- the *target area is defined by the application*.

Motivation

Traditional **LTE-A** *tools* can support these services, but are *unsuitable* to this task because they require too much energy.

For this reason, a new communication framework as been proposed.

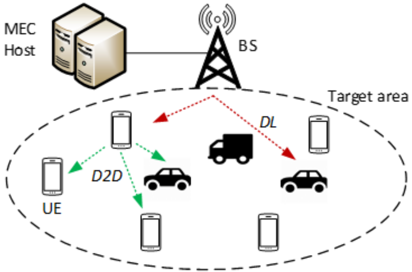

Figure: New communication framework.

Motivation (cont.)

- **eNB** communicates via DL (i.e., vertical) transmissions.
- **UEs** communicate via D2D (i.e. horizontal broadcast) transmissions.
- **Vertical links** are reliable but costly.
- **Horizontal links** are free, but not reliable.
- **UEs** can act as **multi-hop relays** (horizontal transmissions are scheduled by the eNB, which issues grants to the UEs that may transmit).

CCUMRP

- Given the position of the UEs, transmission power, modulation and coding scheme, we can model the probability that a certain horizontal transmission is successful.
- The Chance-Constrained Unicast-Multicast Routing Problem:

CCUMRP

Select vertical and horizontal multi-hop transmissions to guarantee that all UEs receive the information with a certain level of **reliability**, within a given **time limit**, and at **minimum energy** cost.

System Model

We model the system as a **graph** $G = (N, A)$

- $N = \{0\} \cup N'$ (0 is the eNB and N' represents the UEs)
- the arc set $A = A' \cup A''$ consists of two types of arcs:
  - **vertical arcs** A' of the form $(0, i)$ for all $i \in N'$, representing a DL transmission between the eNB the UE i having probability 1 to be decoded successfully at i but high energy cost;
  - **horizontal arcs** A'' of the form (i, j) for $i \neq j \in N'$, representing a D2D transmission from i to j having probability $0 < P_{ij} < 1$ to be decoded successfully at j, but low (energy) cost.

System Model (cont.)

The **problem** is to transmit **from the eNb to the entire floorplan** (i.e., all the UEs).

- **Initial stage:** eNB transmits the message to a subset of UEs using vertical transmission.
- **Following stages:** only horizontal transmissions are allowed.
- A node $i \in N'$ can issue an horizontal transmission at a given stage **only if granted permission** from the eNB.
- **At most** M **grants** can be assigned in each stage.
- The broadcast must be **over in** k **stages.**
- Each UE must receive the message with **probability** $\geq \alpha$.

The **objective** is to minimize the number of vertical transmissions as well as the numbers of grants.

MINLP model

Variables:

- $x_i \in \{0,1\}$, $i \in N'$: **if** node i is **selected** for initial set of UEs at stage 1
- $p_i^h \in [0\,,\,1]$, $i \in N'$, $h \in K$: **probability** of node i at stage h
- $g_i^h \in \{0,1\}$, $i \in N'$, $h \in K'$: **if** node i is **granted** transmission at stage h

$$
\begin{aligned}
\min\ & \textstyle\sum_{i\in N'} x_i + \sum_{h\in K'} \sum_{i\in N'} \beta_i^h g_i^h && && (1)\\
& p_i^1 = x_i && i \in N' && (2)\\
& p_i^k \geq \alpha && i \in N' && (3)\\
& 1 - p_i^h \geq (1 - p_i^{h-1}) \textstyle\prod_{(j,i)\in A''} (1 - g_j^h p_j^{h-1} P_{ji}) && i \in N' \ ,\ h \in K' && (4)\\
& \textstyle\sum_{i\in N'} g_i^h \leq M && h \in K' && (5)\\
& x_i \in \{0,1\} && i \in N' && (6)\\
& 0 \leq p_i^h \leq 1 && i \in N' \ ,\ h \in K && (7)\\
& g_i^h \in \{0,1\} && i \in N' \ ,\ h \in K' && (8)
\end{aligned}
$$

MINLP model (cont.)

- Objective function (1) **minimizes** the number of **vertical transmissions** in the first stage ($h = 1$) and the cost of **grants** in the the subsequent stages ($h \in K'$).
- Constraints (3) impose that each UE node $i \in N'$ is ultimately (at stage k) **reached with probability at least** α.
- Constrains (4) (*nonlinear nonconvex*) represent the **probability** that node i at stage h has **not been reached**.
- Constraints (5) **bound** the total number of **grants** available at each stage.

Without probability constraints (4), the problem would be almost trivial.

Probability constraints (4)

- Take $\log$
- $\implies$ ill-defined when $p_i^h = 1$
- Take a constant $\bar{p} < 1$ "arbitrarily close to 1"
- $\implies$ replace (2) and (7)

$$p_i^1 = x_i \qquad i \in N'$$
$$0 \leq p_i^h \leq 1 \qquad i \in N' \ , \ h \in K'$$

respectively, with:

$$p_i^1 = x_i \bar{p} \qquad i \in N' \tag{9}$$
$$0 \leq p_i^h \leq \bar{p} \qquad i \in N' \ , \ h \in K'. \tag{10}$$

Probability constraints (4) (cont.)

- Note that $g_j^h = 0 \implies \log(1 - g_j^h p_j^{h-1} P_{ji}) = 0$
- $\implies$ rewrite constraints (4) as:

$$\log(1-p_i^h) \geq \log(1-p_i^{h-1}) + \sum_{(j,i)\in A''} g_j^h \log(1-p_j^{h-1} P_{ji}) \quad i \in N' \; h \in K' \backslash \{2\} \tag{11}$$

and

$$\log(1 - p_i^2) \geq \log(1 - \bar{p}) x_i + \sum_{(j,i)\in A''} g_j^2 \log(1 - P_{ji}) x_j \quad i \in N' \quad h = 2 \tag{12}$$

- $\implies$ (11) and (12) are linear with respect to variables g_j^h
- no continuous variables in RHS of (12)

Lagrangian Relaxation

- Relax (11) and (12) with **Lagrangian multipliers** $\lambda_i^h \geq 0$:

$$\min \sum_{i\in N'} x_i + \sum_{i\in N'} \beta_i^2 g_i^2 + \sum_{2\leq h<k} \sum_{i\in N'} \beta_i^{h+1} g_i^{h+1}$$

$$+ \sum_{i\in N'} \lambda_i^2 \left(\log(1-\bar{p})x_i \right.$$

$$\left. + \sum_{i\in N'} \sum_{(j,i)\in A''} g_j^2 \log(1-P_{ji})x_j \right)$$

$$+ \sum_{2\leq h<k} \sum_{i\in N'} (\lambda_i^{h+1} - \lambda_i^h) \log(1-p_i^h)$$

$$+ \sum_{2\leq h<k} \sum_{i\in N'} g_i^{h+1} \sum_{(i,j)\in A''} \lambda_j^{h+1} \log(1-p_i^h P_{ij})$$

$$+ \sum_{i\in N'} -\lambda_i^k \log(1-p_i^k)$$

$$p_i^1 = x_i \qquad i\in N'$$

$$p_i^k \geq \alpha \qquad i\in N'$$

$$\sum_{i\in N'} g_i^h \leq M \qquad h\in K'$$

$$x_i \in \{0,1\} \qquad i\in N'$$

$$0 \leq p_i^h \leq 1 \qquad i\in N', \quad h\in K$$

$$g_i^h \in \{0,1\} \qquad i\in N', \quad h\in K'$$

- $\Longrightarrow$ problem **decomposes** into k **sub-problems**

Sub-problem $h = 1$

- The **first sub-problem** contains variables x_i and $g_i^2,\ i \in N'$.
- Collect "like terms".
- Observe that there is no point in setting $g_i^2 = 1$ if $x_i = 0$.

$$
\begin{aligned}
\min\ & \textstyle\sum_{i \in N'} \left[(1 + \lambda_i^2 \log(1 - \bar{p}))x_i + \left(\beta_i^2 + \sum_{(i,j) \in A''} \lambda_j^2 \log(1 - P_{ij})\right) g_i^2\right] \\
& \textstyle\sum_{i \in N'} g_i^2 \le M \\
& g_i^2 \le x_i \qquad\qquad i \in N' \\
& x_i,\ g_i^2 \in \{0, 1\} \qquad\qquad i \in N'
\end{aligned}
$$

- Nonlinear operations are applied to constants $\rightarrow$ $\{0, 1\}$ LP.
- Special structure of the constraints $\rightarrow$ solved in $\mathcal{O}(n \log n)$.

Sub-problems $2 \leq h < k$

- Each h-**th sub-problem** contains variables g_i^{h+1} and p_i^h, $i \in N'$.

$$\min \sum_{i \in N'} \big[(\lambda_i^{h+1} - \lambda_i^h) \log(1 - p_i^h) + \big(\beta_i^{h+1} + \sum_{(i,j) \in A''} \lambda_j^{h+1} \log(1 - p_i^h P_{ij})\big) g_i^{h+1} \big] \quad (13)$$

$$0 \leq p_i^h \leq \bar{p} \qquad i \in N'$$

$$\sum_{i \in N'} g_i^{h+1} \leq M \quad (14)$$

$$g_i^{h+1} \in \{0, 1\} \qquad i \in N' \quad (15)$$

- Each variable p_i^h interacts with a variable g_i^{h+1}.
- The term is highly nonlinear: $f_i^h(p, g) = (\lambda_i^{h+1} - \lambda_i^h) \log(1 - p) + \big(\beta_i^{h+1} + \sum_{(i,j) \in A''} \lambda_j^{h+1} \log(1 - pP_{ij})\big) g.$

Sub-problems $2 \leq h < k$

- By computing the two constants for $g \in \{0, 1\}$:

$$p_i^{h,g} = \text{argmin}\{ f_i^h(p, g) : 0 \leq p \leq \bar{p} \}$$

- the **sub-problem** h can be rewritten as

$$\min \{ \textstyle\sum_{i \in N'} f_i^h(p_i^{h,1}, 1) g_i^{h+1} + f_i^h(p_i^{h,0}, 0)(1 - g_i^{h+1}) : (14) , (15) \}$$

- $\Longrightarrow$ easily solved in $\mathcal{O}(n \log n)$.

- Computing $p_i^{h,0}$ is trivial: minimize the monotone function $(\lambda_i^{h+1} - \lambda_i^h) \log(1 - p)$ on $p \in [0, \bar{p}] \Longrightarrow$ the optimum is one of the two extremes.

- Computing $p_i^{h,1}$ requires to solve a more complex one-dimensional minimization problem of the form:

$$\min \{ f(p) = c \log(1 - p) + \textstyle\sum_{i \in N'} a_i \log(1 - pb_i) : 0 \leq p \leq \bar{p} \} \quad (16)$$

- Use a simple *globalization of Newton's method*.

Sub-problem $h = k$

- The **k-th sub-problem** contains variables p_i^k, $i \in N'$.

$$\min \left\{ \sum_{i \in N'} -\lambda_i^k \log(1 - p_i^k) \ : \ \alpha \le p_i^k \le \bar{p} \right\}$$

- Separable over i.
- $\lambda_i^k \ge 0 \implies$ objective function is convex.
- $\implies$ the optimum is the left endpoint $p_i^k = \alpha$.

Solution method

- To solve the Lagrangian Dual problem we use the freely available implementation of the (generailized) *proximal Bundle method*, provided by the `NDOSolver/FiOracle` suite.
- Lagrangian heuristics
- Lagrangian-based branch-and-bound

Lagrangian heuristics

- Use integer, but (typically) not feasible solution from the Lagrangian function.
- Note that continuous variables p_j^h can be computed from the x_i and g_i^h.
- If the relaxed constraints (3) are violated $\implies$ not enough transmission to ensure probability "at least" α of reception.
- Given an integer solution, we define the scores as follows:

$$S(x_i) = \sum_{j\,:\,p_j^k<\alpha} P_{i,j}^2(\alpha - p_j^k)^3\delta_i$$
$$S(g_i^h) = \sum_{j\,:\,p_j^k<\alpha} P_{i,j}^2(\alpha - p_j^k)^3 p_i^{h-1}$$

- The higher the connection $P_{i,j}$ with users that did not receive the message $\implies$ the higher the term $(\alpha - p_j^k)$.

Lagrangian heuristics (cont.)

- x_i variables have a much larger cost than the g_i^h ones $\implies$ scaling factor $\delta_i < 1$.
- The exponents of these factors can be changed to give more or less impact in the final score.
- Greedy: until we reach a feasible solution, we activate the variable with best score that does not violate (5).
- The algorithm can start from any integer solution satisfying (5) (e.g., the solution of the Lagrangian relaxation, the null solution).
- Another heuristic is based on the convexified solution obtained from the Lagrangian dual.
- This is a fractional solution, that can be rounded in order to find an integer one.

Lagrangian-based Branch-and-Bound

- At each node, solve Lagrangian Dual with time limit of 1 sec.
- Run the heuristics based on integer solution.
- Branching rule based on $0/1$ fixing of a binary variable.
- Look at the convexified solution and pick fractional x variable that has the closest value to 0.5.
- If all the x variables are already fixed, or take integer values in the convexified solution, choose a g variable with a similar rule.
- Branching first on the x variables makes sense since they are those that are likely to have the most impact on the solution of the problem.
- Visit strategy is the standard "best first": we pick the node that have the lowest lower bound.

Experiments

- Realistic scenarios generated via simulator `SimuLTE`.
- Instance:
  - number of UEs
  - radius (in meters) of the area
  - required coverage probability α
- Comparison with `BARON`, and a combinatorial heuristic available in `SimuLTE`.
- Compiled with g++ 7.4.0, single-thread, 16-core Intel Xeon5120 CPU, 2.20GHz, 64Gb RAM, Ubuntu 18.04.

Experiments: $\# \text{ UEs} = 10$

Instances			BARON					B&B					CH
#	r	α	time	nodes	gap	pgap	dgap	time	nodes	gap	pgap	dgap	pgap
10	100	0.92	4.86	1	0.00	0.00	0.00	0.59	20	0.00	0.00	0.00	0.00
10	100	0.95	3.07	1	0.00	0.00	0.00	0.58	20	0.00	0.00	0.00	0.00
10	100	0.96	3.43	1	0.00	0.00	0.00	0.67	20	0.00	0.00	0.00	0.00
10	250	0.92	4.92	1	0.00	0.00	0.00	0.44	20	0.00	0.00	0.00	0.00
10	250	0.95	75.39	1	0.00	0.00	0.00	0.73	20	0.00	0.00	0.00	0.00
10	250	0.96	31.32	1	0.00	0.00	0.00	0.46	20	0.00	0.00	0.00	71.4
10	500	0.92	80.67	84	0.00	0.00	0.00	193.8	12323	0.00	0.00	0.00	71.4
10	500	0.95	44.45	52	0.00	0.00	0.00	44.00	2717	0.00	0.00	0.00	40.0
10	500	0.96	383.4	1597	0.00	0.00	0.00	229.1	5130	0.00	0.00	0.00	46.7
10	750	0.92	269.2	1778	0.00	0.00	0.00	153.42	2402	0.00	0.00	0.00	29.4
10	750	0.95	–	715	4.00	0.00	4.00	208.5	6880	0.00	0.00	0.00	38.5
10	750	0.96	–	717	13.0	0.00	13.0	29.87	1026	0.00	0.00	0.00	50.0
10	1000	0.92	1.78	1	0.00	0.00	0.00	79.81	2913	0.00	0.00	0.00	26.9
10	1000	0.95	1.42	1	0.00	0.00	0.00	210.0	13754	0.00	0.00	0.00	36.4
10	1000	0.96	0.82	1	0.00	0.00	0.00	1.91	120	0.00	0.00	0.00	63.6

Table: Computational results, time limit 300 seconds

Experiments: # UEs = 25

Instances			BARON					B&B					CH
#	r	α	time	nodes	gap	pgap	dgap	time	nodes	gap	pgap	dgap	pgap
25	100	0.92	–	1	3780	3050	120	121.6	164	0.00	0.00	0.00	0.00
25	100	0.95	–	17	100	0.00	100	98.45	130	0.00	0.00	0.00	0.00
25	100	0.96	–	18	80.0	0.00	80	57.56	84	0.00	0.00	0.00	0.00
25	250	0.92	–	1	3780	3050	120	12.83	58	0.00	0.00	0.00	0.00
25	250	0.95	–	1	3780	2600	140	11.30	58	0.00	0.00	0.00	0.00
25	250	0.96	–	1	3780	2600	140	10.04	56	0.00	0.00	0.00	0.00
25	500	0.92	–	1	3780	1354	260	–	1995	40.0	7.69	30.0	30.8
25	500	0.95	–	1	3780	1354	260	–	1983	23.1	23.1	0.00	69.2
25	500	0.96	–	1	3780	1250	260	–	1569	23.1	14.3	0.00	42.9
25	750	0.92	–	2	1160	456	80	–	642	40.0	2.94	8.00	32.4
25	750	0.95	–	5	1081	425	88	–	1263	23.3	2.78	0.00	22.2
25	750	0.96	–	4	1081	425	88	–	1004	23.3	2.78	0.00	22.2
25	1000	0.92	–	12	330	210	25	–	1332	14.6	3.28	0.00	36.2
25	1000	0.95	–	10	311	205	26	–	1185	12.5	1.61	3.57	29.0
25	1000	0.96	–	12	294	200	25	–	951	12.3	1.59	5.26	47.6

Table: Computational results, time limit 300 seconds

Experiments: # UEs = 50

Instances			BARON					B&B					CH
#	r	α	time	nodes	gap	pgap	dgap	time	nodes	gap	pgap	dgap	pgap
50	100	0.92	–	1	6280	5133	40	–	283	100	0.00	20.0	0.00
50	100	0.95	–	1	6280	5133	40	–	283	100	0.00	20.0	0.00
50	100	0.96	–	1	6280	5133	40	–	283	100	0.00	20.0	0.00
50	250	0.92	–	1	780	550	80	–	283	60.0	0.00	20.0	0.00
50	250	0.95	–	1	6280	4386	80	–	283	80.0	0.00	20.0	0.00
50	250	0.96	–	1	6280	4386	80	–	284	80.0	0.00	20.0	0.00
50	500	0.92	–	1	6280	2143	140	–	283	180	0.00	40.0	21.4
50	500	0.95	–	1	6280	1993	160	–	284	114	0.00	14.3	20.0
50	500	0.96	–	1	6280	1993	160	–	283	87.5	0.00	0.00	20.0
50	750	0.92	–	1	6280	913	200	–	292	230	6.45	0.00	3.20
50	750	0.95	–	1	6280	772	260	–	291	192	5.56	0.00	22.2
50	750	0.96	–	1	6280	749	260	–	290	185	0.00	0.00	18.9
50	1000	0.92	–	1	6280	398	560	–	283	220	1.59	40.0	11.1
50	1000	0.95	–	1	6280	376	600	–	280	156	4.55	11.1	28.8
50	1000	0.96	–	1	6280	355	700	–	280	154	2.90	25.0	31.9

Table: Computational results, time limit 300 seconds

Experiments: $\#$ UEs $= 10$

Instances			BARON					B&B					CH
$\#$	r	α	time	nodes	gap	pgap	dgap	time	nodes	gap	pgap	dgap	pgap
10	100	0.92	5.13	1	0.00	0.00	0.00	0.59	20	0.00	0.00	0.00	0.00
10	100	0.95	3.12	1	0.00	0.00	0.00	0.58	20	0.00	0.00	0.00	0.00
10	100	0.96	3.48	1	0.00	0.00	0.00	0.67	20	0.00	0.00	0.00	0.00
10	250	0.92	4.69	1	0.00	0.00	0.00	0.44	20	0.00	0.00	0.00	0.00
10	250	0.95	75.22	1	0.00	0.00	0.00	0.73	20	0.00	0.00	0.00	0.00
10	250	0.96	31.12	1	0.00	0.00	0.00	0.46	20	0.00	0.00	0.00	71.4
10	500	0.92	82.39	84	0.00	0.00	0.00	193.7	12323	0.00	0.00	0.00	71.4
10	500	0.95	46.08	52	0.00	0.00	0.00	44.0	2717	0.00	0.00	0.00	40.0
10	500	0.96	383.4	1597	0.00	0.00	0.00	229.1	5130	0.00	0.00	0.00	46.7
10	750	0.92	269.2	1778	0.00	0.00	0.00	153.42	2402	0.00	0.00	0.00	29.4
10	750	0.95	439.0	911	0.00	0.00	0.00	208.5	6880	0.00	0.00	0.00	38.5
10	750	0.96	1456	2605	0.00	0.00	0.00	29.9	1026	0.00	0.00	0.00	50.0
10	1000	0.92	1.78	1	0.00	0.00	0.00	79.81	2913	0.00	0.00	0.00	26.9
10	1000	0.95	1.66	1	0.00	0.00	0.00	210.0	13754	0.00	0.00	0.00	36.4
10	1000	0.96	0.90	1	0.00	0.00	0.00	1.91	120	0.00	0.00	0.00	63.6

Table: Computational results, time limit 3000 seconds

Experiments: # UEs = 25

Instances			BARON					B&B					CH
#	r	α	time	nodes	gap	pgap	dgap	time	nodes	gap	pgap	dgap	pgap
25	100	0.92	–	71	100	0.00	100	121.6	164	0.00	0.00	0.00	0.00
25	100	0.95	–	94	80.0	0.00	80.0	98.5	130	0.00	0.00	0.00	0.00
25	100	0.96	–	109	80.0	0.00	80.0	57.6	84	0.00	0.00	0.00	0.00
25	250	0.92	–	107	80.0	0.00	80.0	12.8	58	0.00	0.00	0.00	0.00
25	250	0.95	–	21	100	0.00	100	11.3	58	0.00	0.00	0.00	0.00
25	250	0.96	–	18	100	0.00	100	10.0	56	0.00	0.00	0.00	0.00
25	500	0.92	–	29	3680	1354	160	–	5300	27.3	7.69	18.2	30.8
25	500	0.95	–	28	3050	1354	117	–	7552	7.69	7.69	0.00	69.2
25	500	0.96	–	39	3050	1250	117	–	6404	23.1	14.3	0.00	42.9
25	750	0.92	–	35	950	456	50.0	–	4295	34.6	2.94	3.85	32.4
25	750	0.95	–	52	845	425	50.0	–	8314	23.3	2.78	0.00	22.2
25	750	0.96	–	49	845	425	50.0	–	4485	26.7	5.56	0.00	22.2
25	1000	0.92	–	82	294	210	14.6	–	12406	12.7	1.64	0.00	36.1
25	1000	0.95	–	83	286	205	18.4	–	11378	10.5	1.61	1.75	29.0
25	1000	0.96	–	104	49.0	20.6	17.7	–	10330	6.67	1.59	0.00	47.6

Table: Computational results, time limit 3000 seconds

Experiments: # UEs = 50

Instances			BARON					B&B					CH
#	r	α	time	nodes	gap	pgap	dgap	time	nodes	gap	pgap	dgap	pgap
50	100	0.92	–	11	100	0.00	20	–	2805	80.0	0.00	0.00	0.00
50	100	0.95	–	1	6280	5133	40	–	2804	80.0	0.00	0.00	0.00
50	100	0.96	–	1	6280	5133	40	–	2803	80.0	0.00	0.00	0.00
50	250	0.92	–	1	80.0	0.00	40	–	2795	40.0	0.00	0.00	0.00
50	250	0.95	–	1	100	0.00	40	–	2796	60.0	0.00	0.00	0.00
50	250	0.96	–	1	100	0.00	40	–	2794	60.0	0.00	0.00	0.00
50	500	0.92	–	1	6280	2143	140	–	2773	100	0.00	0.00	21.4
50	500	0.95	–	1	6280	1993	160	–	2779	87.5	0.00	0.00	20.0
50	500	0.96	–	1	6280	1993	160	–	2763	87.5	0.00	0.00	20.0
50	750	0.92	–	8	3040	913	0.00	–	2947	230	6.45	0.00	3.20
50	750	0.95	–	10	3040	773	30.0	–	2942	177	0.00	0.00	22.2
50	750	0.96	–	8	3040	749	30.0	–	2955	185	0.00	0.00	18.9
50	1000	0.92	–	20	1470	398	40.0	–	2825	125	0.00	0.00	11.1
50	1000	0.95	–	13	1395	376	42.9	–	2880	127	3.03	0.00	28.8
50	1000	0.96	–	14	1327	355	59.1	–	2784	97.1	0.00	0.00	31.9

Table: Computational results, time limit 3000 seconds

Thank you!