Start-up/Shut-down MINLP formulations for the Unit Commitment with Ramp Constraints

Tiziano Bacci

joint work with A. Frangioni and C. Gentile

IASI-CNR

tiziano.bacci@iasi.cnr.it

CTW 2020

14-16 September 2020

Index

Unit Commitment Problem (UC): definition

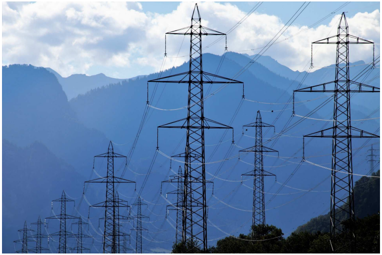
Given a set of power units and a time horizon, determine at any time period

- whether a unit is producing energy
- the amount of energy production for each unit

while minimizing the production costs and satisfying all the constraints

Type of units:

- **Thermal**
- Hydroelectric
- Renewable Energy Sources (wind, solar, . . .)



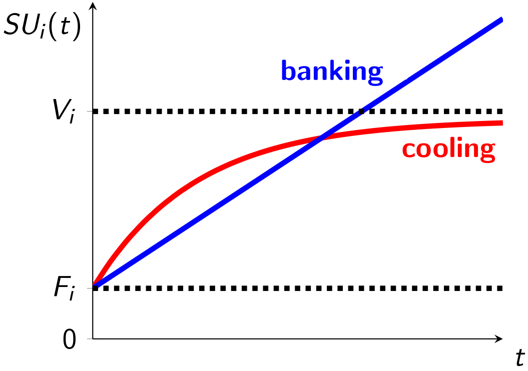
Unit Commitment Problem (UC): objective function

Minimization of the total production cost including:

- Generation costs

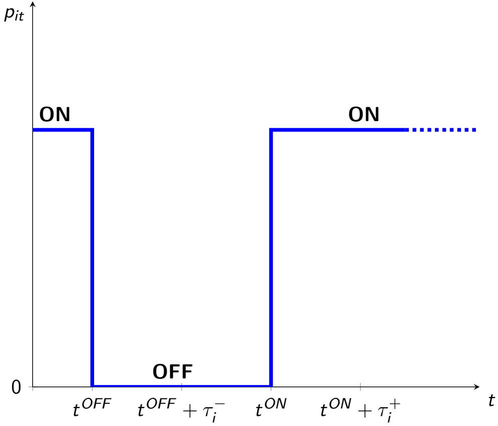
  $f^i(p_{it}) = a_i p_{it}^2 + b_i p_{it}$ (plus possibly a fixed cost c_i), for each unit i and time period t

- Start-Up costs



Unit Commitment Problem (UC): constraints

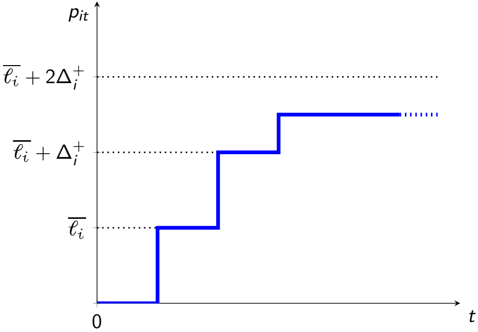
- Min/Max power output
  $\ell_i \leq p_{it} \leq u_i$ (ℓ_i = min. power, u_i = max. power unit i)
- Minimum Up/Down-time



τ_i^+ = min. time in ON state, τ_i^- = min. time in OFF state unit i

Unit Commitment Problem (UC): constraints

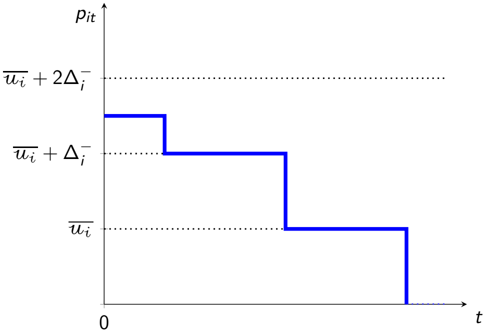
- Ramp Up limits



$\overline{\ell_i}$ = start-up limit, Δ_i^+ = ramp-up limit unit i

Unit Commitment Problem (UC): constraints

- Ramp Down limits



$\overline{u_i}$ = shut-down limit, Δ_i^- = ramp-down limit unit i

- Energy Demand

$\sum_{i \in \mathcal{I}} p_{it} = d_t$, for each time period t ($\mathcal{I}$:= set of units)

MINLP Formulations: 3-bin[1]

- Variables 3-bin formulation:

  p_{it} = power production of unit i at time t

  $x_{it} = 1$ if unit i is committed at time t, 0 o/w

  $v_{it} = 1$ if unit i has been started up at time t, 0 o/w

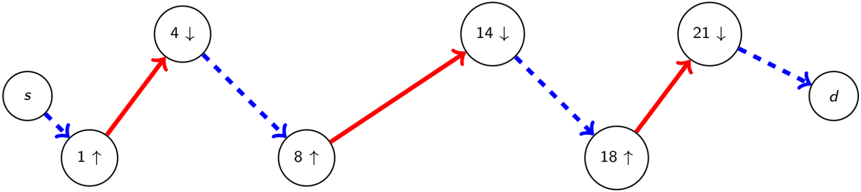
  $w_{it} = 1$ if unit i has been shut down at time t, 0 o/w

---

[1] Rajan, Takriti, "Minimum Up/Down polytopes of the unit commitment problem with start-up costs", IBM RC23628, 2005

MINLP Formulations: Dynamic Programming based[3]

- 3 different models: **DP**, $\boldsymbol{p_t}$ and **SU**
- Derived from DP algorithm[2] based on the state-space graph
- nodes $(t,\uparrow)/(t,\downarrow)$: unit starts up/shuts down at time t
- An *s*-*d* path from represents a schedule for the unit



- "on" arcs $(\,(h,\uparrow)\,,\,(k,\downarrow)\,)$: generation costs
- "off" arcs $(\,(h,\downarrow)\,,\,(k,\uparrow)\,)$: start-up costs

---

[2] Frangioni, Gentile "Solving Nonlinear Single-Unit Commitment Problems with Ramping Constraints" *Op. Res.*, 2006

[3] Bacci, Frangioni, Gentile Tavlaridis-Gyparakis "New MI-SOCP Formulations for the Single-Unit [. . .]", IASI RR, 2019

MINLP Formulations: Dynamic Programming based

- 3 different models: **DP**, $\boldsymbol{p_t}$ and **SU**
- Variables common to the 3 models (binary)

  $y_{i,\text{ON}}^{h,k} = 1$ if unit i starts-up in h and shuts-down in k, 0 o/w

  $y_{i,\text{OFF}}^{h,k} = 1$ if unit i shuts-down in h and starts-up in k, 0 o/w
- Specific variables for each models (continuous)

  $p_{it}^{hk} =$ power of unit i at time t if starts-up in h and shuts-down in k (**DP** model)

  $p_{it} =$ power of unit i at time t ($\boldsymbol{p_t}$ model)

  $p_{it}^{h} =$ power of unit i at time t if starts-up in h (**SU** model)

MINLP Formulations: Perspective reformulation[4]

- Objective function $f^i(p_{it}) = a_i p_{it}^2 + b_i p_{it}$, for each unit i and time period t is convex but p_{it} are semi-continuos
- “Real” Objective function is not convex

$$f^i(p_{it}, x_{it}) = \begin{cases} a_i p_{it}^2 + b_i p_{it} & \text{if } \ell_i \leq p_{it} \leq u_i \text{ and } x_{it} = 1 \\ 0 & \text{if } p_{it} = x_{it} = 0 \\ \infty & \text{otherwise} \end{cases}$$

- Convex with Perspective reformulation

$$h^i(p_{it}, x_{it}) = \begin{cases} a_i \frac{p_{it}^2}{x_{it}} + b_i p_{it} & \text{if } x_{it} > 0 \\ 0 & \text{if } x_{it} = 0. \end{cases}$$

- $h^i(p_{it}, x_{it}) > f^i(p_{it})$ if $0 < x_{it} < 1$,
  $h^i(p_{it}, x_{it}) = f^i(p_{it})$ if $x_{it} \in \{0, 1\}$

---

[4] Frangioni, Gentile, “Perspective cuts for a class of convex 0-1 mixed integer programs”, *Math. Prog.*, 2006

MINLP Formulations: Perspective reformulation

- Objective functions:

- **3bin**: $x_{it} f^i(p_{it}/x_{it}) + c_{it} x_{it}$, for each unit i and time t

- **DP**: $y^{h,k}_{i,\mathrm{ON}} f^i(p^{hk}_{it}/y^{h,k}_{i,\mathrm{ON}}) + c^{hk}_{it} y^{h,k}_{i,\mathrm{ON}}$, for each unit i and time t and each start-up h and shut-down k

- $\boldsymbol{p_t}$: $\sum_{(h,k)\,:h\leq t\leq k} y^{h,k}_{i,\mathrm{ON}} f^i(p_{it}/\sum_{(h,k)\,:h\leq t\leq k} y^{h,k}_{i,\mathrm{ON}}) + c_{it} \sum_{(h,k)\,:h\leq t\leq k} y^{h,k}_{i,\mathrm{ON}}$, for each unit i and time t

- **SU**: $\sum_{k\,:t\leq k} y^{h,k}_{i,\mathrm{ON}} f^i(p^h_{it}/\sum_{k\,:t\leq k} y^{h,k}_{i,\mathrm{ON}}) + c^h_{it} \sum_{k\,:t\leq k} y^{h,k}_{i,\mathrm{ON}}$, for each unit i and time t and each start-up h

New MINLP formulations

- Two new DP based formulations for the UC
- ⋆ **SD** formulation: it is a nearly symmetric of the SU model
- ⋆ **SUSD** formulation: it is a combination of the SU and the SD models
- Extensive computational experiments for testing the two new models

New MINLP formulations: the SD model

- A nearly symmetric version of the SU model

  Variables

  $y_{i,\text{ON}}^{h,k} = 1$ if unit i starts-up in h and shuts-down in k, 0 o/w
  $y_{i,\text{OFF}}^{h,k} = 1$ if unit i shuts-down in h and starts-up in k, 0 o/w

  $\tilde{p}_{it}^{k} =$ power of unit i at time t if shuts-down in k

  Connection between p_t, SU and SD models

  $p_{it} = \sum_{h\,:h \leq t} p_{it}^{h} = \sum_{k\,:t \leq k} \tilde{p}_{it}^{k}$
  (p_t) (SU) (SD)

New MINLP formulations: the SUSD model

- It is a combination of the SU and the SD models

  Variables

  $y_{i,\mathrm{ON}}^{h,k} = 1$ if unit i starts-up in h and shuts-down in k, 0 o/w
  $y_{i,\mathrm{OFF}}^{h,k} = 1$ if unit i shuts-down in h and starts-up in k, 0 o/w

  $p_{it}^{h} =$ power of unit i at time t if starts-up in h (from **SU**)
  $\tilde{p_{it}^{k}} =$ power of unit i at time t if shuts-down in k (from **SD**)

$$\theta_{it} := \max \left\{ \begin{array}{l} \sum_{h\,:h\leq t}(\sum_{k\,:t\leq k} y_i^{hk}) f^i(p_{it}^{h}/(\sum_{k\,:t\leq k} y_i^{hk})) \rightarrow (SU) \\ \\ \sum_{k\,:t\leq k}(\sum_{h\,:h\leq t} y_i^{hk}) f^i(\tilde{p_{it}^{k}}/(\sum_{h\,:h\leq t} y_i^{hk})) \rightarrow (SD) \end{array} \right\}$$

MINLP formulations sizes

Bounds on the number of variables and constraints for each model

Model	Power variables	Other variables	Constraints
3-bin	$O(n)$	$O(n)$	$O(n)$
DP	$O(n^3)$	$O(n^2)$	$O(n^3)$
p_t	$O(n)$	$O(n^2)$	$O(n^2)$
SU SD SUSD	$O(n^2)$	$O(n^2)$	$O(n^2)$

Computational results: sizes compared to the 3-bin

	DP		p_t		SU		SD		SUSD	
units	vars	cons	vars	cons	vars	cons	vars	cons	vars	cons
10	20	38	2	1	4	6	6	8	8	14
20	23	44	2	1	5	6	6	8	9	14
50	23	45	2	1	5	6	6	8	9	15

Ratio between number of variables and constraints w.r.t. the 3-bin model

Computational results: LP gaps

	3-bin		DP		p_t	
units	time	gap	time	gap	time	gap
10	0.15	2.077	16.81	1.222	0.94	1.335
20	0.34	1.691	93.61	0.811	2.21	0.894
50	1.19	0.906	574.98	0.137	6.88	0.190

	SU		SD		SUSD	
units	time	gap	time	gap	time	gap
10	2.85	1.280	3.94	1.264	46.26	1.222
20	8.40	0.822	12.34	0.880	161.70	0.811
50	34.06	0.146	42.98	0.178	772.12	0.137

Computing times and gaps of the linear relaxation for each model

Computational results: experiments with gap 10^{-4}

	3-bin				DP				p_t			
units	time	opt	nodes	gap	time	opt	nodes	gap	time	opt	nodes	gap
10	83	10	407	0.01	1182	10	568	0.01	121	10	267	0.01
20	7138	4	2728	0.10	6526	6	1147	0.23	1888	9	1088	0.06
50	10000	0	1469	0.17	10001	0	473	0.49	9025	1	1538	0.06

	SU				SD				SUSD			
units	time	opt	nodes	gap	time	opt	nodes	gap	time	opt	nodes	gap
10	206	10	352	0.00	234	10	359	0.01	1173	10	549	0.01
20	3000	8	1642	0.10	4405	9	1780	0.01	8677	4	1080	0.56
50	9053	1	1296	0.08	8528	2	1244	0.23	10000	0	393	0.36

Computing times, explored nodes, number of instances solved over 10 (time limit 10000 seconds)

Conclusions

- Two new MINLP formulations for the UC: the SU and SUSD models
- DP based models are a trade-off between size and good bounds
- SD model performs better than the SU one
- SUSD model gives value of the LP very near to the DP one

⋆ Future research

Column-and-Row generation approach could be effective for DP based models

Thank you for your attention

Computational results: sizes before and after pre-solve

	3-bin				DP				p_t			
units	v	c	%v	%c	v	c	%v	%c	v	c	%v	%c
10	2e+3	2e+3	23	20	3e+4	8e+4	12	22	3e+3	2e+3	13	14
20	3e+3	4e+3	21	19	7e+4	2e+5	11	19	8e+3	4e+3	12	13
50	8e+3	1e+4	19	17	2e+5	5e+5	11	20	2e+4	1e+4	8	11

	SU				SD				SUSD			
units	v	c	%v	%c	v	c	%v	%c	v	c	%v	%c
10	7e+3	1e+4	13	19	9e+3	2e+4	9	9	1e+4	3e+4	9	13
20	2e+4	3e+4	12	19	2e+4	3e+4	8	9	3e+4	6e+4	8	13
50	4e+4	7e+4	9	13	5e+4	9e+4	8	9	7e+4	2e+5	6	10

LP size before CPLEX presolve and percentage of reduction after presolve

Computational results: experiments with gap 10^{-3}

	3-bin				DP				p_t			
units	time	opt	nodes	gap	time	opt	nodes	gap	time	opt	nodes	gap
10	77	10	333	0.08	1150	10	477	0.09	54	10	175	0.07
20	6213	5	1914	0.14	5859	7	824	0.28	1413	9	623	0.14
50	7038	3	909	0.20	8671	3	402	0.48	3409	7	434	0.10

	SU				SD				SUSD			
units	time	opt	nodes	gap	time	opt	nodes	gap	time	opt	nodes	gap
10	199	10	299	0.09	222	10	297	0.07	1103	10	463	0.08
20	2366	9	1088	0.15	3255	10	1132	0.09	7068	5	692	0.62
50	4978	7	748	0.10	6248	7	735	0.25	9050	1	320	0.36

Computing times, explored nodes, number of instances solved over 10 (time limit 10000 seconds)

Max. power constraints (example)

Max power constraints for each model (u_i = max. power unit i)

- $p_{it} \leq u_i x_{it}$ (3-bin)
- $p_{it}^{hk} \leq u_i y_{i,\mathrm{ON}}^{h,k}$ (DP)
- $p_{it} \leq u_i \sum_{(h,k)\,:h \leq t \leq k} y_{i,\mathrm{ON}}^{h,k}$ (p_t)
- $p_{it}^{h} \leq u_i \sum_{h\,:h \leq t} y_{i,\mathrm{ON}}^{h,k}$ (SU)
- $\tilde{p}_{it}^{k} \leq u_i \sum_{k\,:t \leq k} y_{i,\mathrm{ON}}^{h,k}$ (SD)

Recall:

- $x_{it} = 1$ if unit i is committed at time t, 0 o/w (3-bin)
- $y_{i,\mathrm{ON}}^{h,k} = 1$ if unit i starts-up in h and shuts-down in k, 0 o/w (DP)

Max. power constraints (example)

Max power constraints for each model (u_i = max. power unit i)

- $p_{it} \leq u_i x_{it}$ (3-bin)
- $p_{it}^{hk} \leq u_i y_{i,\mathrm{ON}}^{h,k}$ (DP)
- $p_{it} \leq u_i \sum_{(h,k)\,:h \leq t \leq k} y_{i,\mathrm{ON}}^{h,k}$ (p_t)
- $p_{it}^{h} \leq u_i \sum_{h\,:h \leq t} y_{i,\mathrm{ON}}^{h,k}$ (SU)
- $\tilde{p}_{it}^{k} \leq u_i \sum_{k\,:t \leq k} y_{i,\mathrm{ON}}^{h,k}$ (SD)
- $\sum_{h\,:h \leq t} p_{it}^{h} = \sum_{k\,:t \leq k} \tilde{p}_{it}^{k}$

(SUSD)