Synchronized Pickup and Delivery Problems with Connecting FIFO Stack

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  - **Pickup network** (item storage)
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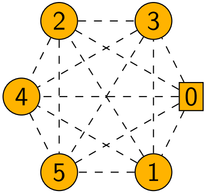

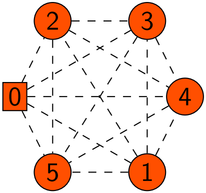

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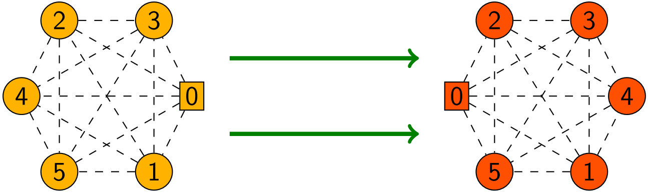

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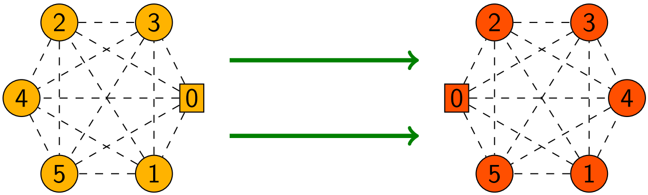

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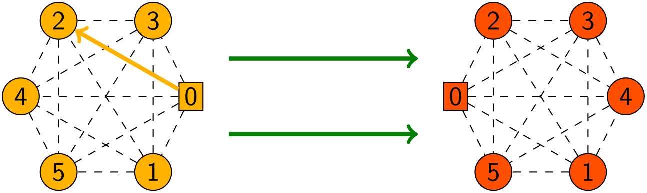

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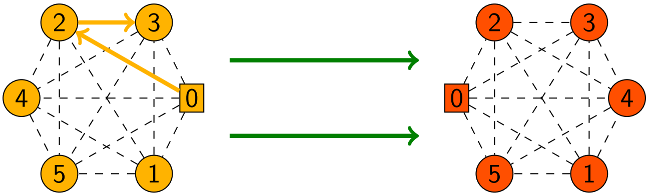

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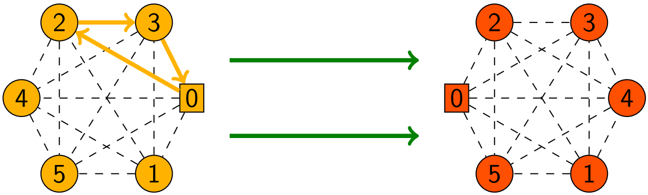

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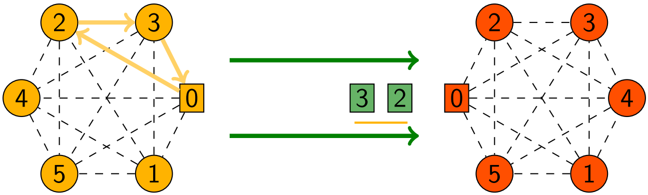

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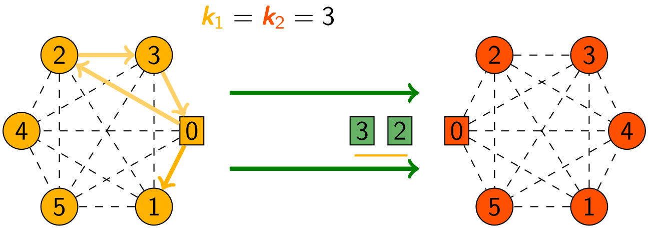

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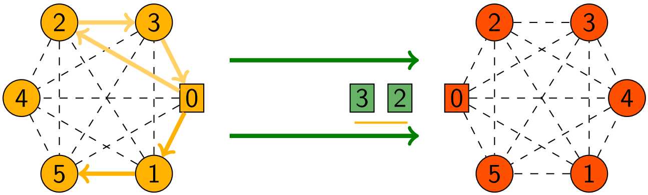

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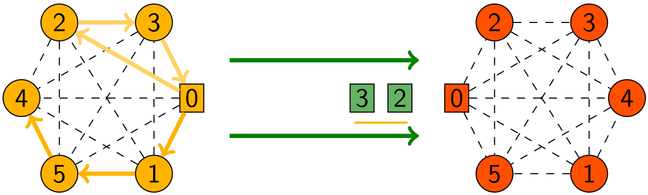

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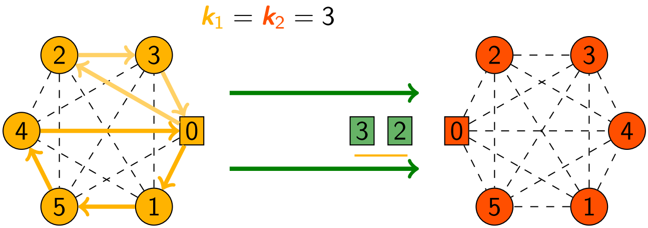

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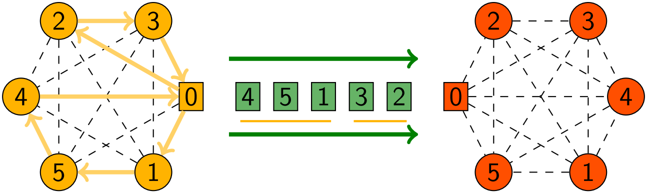

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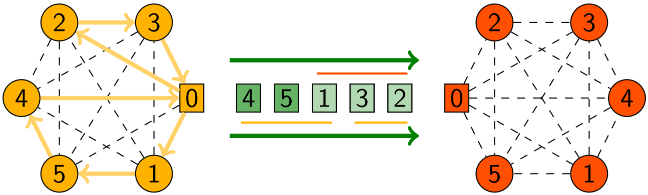

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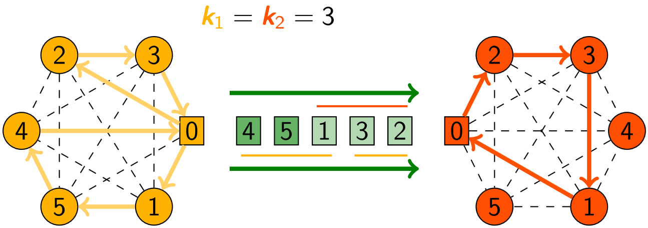

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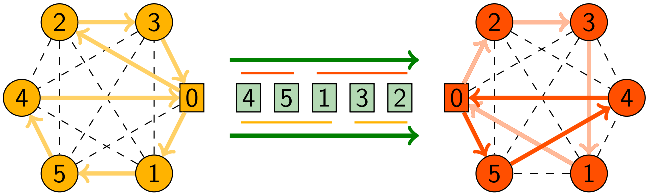

Notation and Definitions

Basic data

- $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{A})$ complete digraph
- $\boldsymbol{c}^1, \boldsymbol{c}^2 \colon \boldsymbol{A} \to \mathbb{R}_+$ **cost** functions
- $\boldsymbol{k}_1, \boldsymbol{k}_2$ vehicle **capacities**

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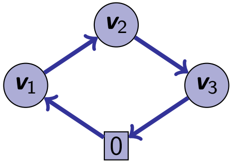
  - $v_j \neq 0$ for all $j = 1, 2, \ldots, k$
  - for a trip in D^i:
    - **feasibility:** $k \leq k_i$
    - **trip cost:** $c^i(t) = c^i(0, v_1) + \sum_{i=1}^{k-1} c^i(v_i, v_{i+1}) + c^i(v_k, 0)$



A General Problem Definition

A **Synchronized Pickup and Delivery Problem with FIFO stack** (**SPDP-FS**) is

$$
\begin{aligned}
\min \quad & \boldsymbol{c}^1(\boldsymbol{P}) + \boldsymbol{c}^2(\boldsymbol{D}) \\
\text{s. t.} \quad & \\
& \boldsymbol{P} = (\boldsymbol{p}_1, \boldsymbol{p}_2, \dots, \boldsymbol{p}_\ell) \qquad \text{with } \boldsymbol{p_i} \text{ feasible trips partitioning } \boldsymbol{V} \setminus 0 \\
& \boldsymbol{D} = (\boldsymbol{d}_1, \boldsymbol{d}_2, \dots, \boldsymbol{d_m}) \qquad \text{with } \boldsymbol{d_j} \text{ feasible trips partitioning } \boldsymbol{V} \setminus 0 \\
& (\boldsymbol{P}, \boldsymbol{D}) \text{ satisfies the } \textbf{FIFO}
\end{aligned}
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No-Permutation Description:

- items on the **FIFO** stack respecting the **pickup order**
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Example

- $\boldsymbol{T} = ((2,3),(1,5,4))$
- **$\boldsymbol{T}$-sequence**$= (2,3,1,5,4)$

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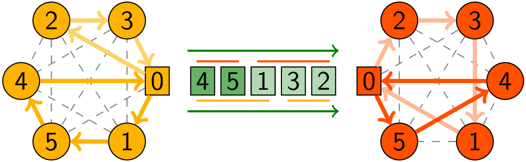

The Permutation Variants

Permutation Description

- Each pickup trip unloads a **batch** of items on the **FIFO stack**
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- Each **delivery batch** is **contained** in one **pickup batch**
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SPDP-FS with No-Overlap

$(\boldsymbol{P}, \boldsymbol{D})$ satisfies the **requirement** **if** for all $\boldsymbol{i}$ there is $\boldsymbol{j}$ s.t. $\boldsymbol{V}(\boldsymbol{d}_{\boldsymbol{i}}) \subseteq \boldsymbol{V}(\boldsymbol{p}_{\boldsymbol{j}})$

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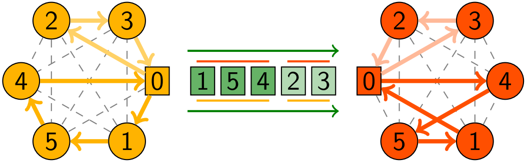
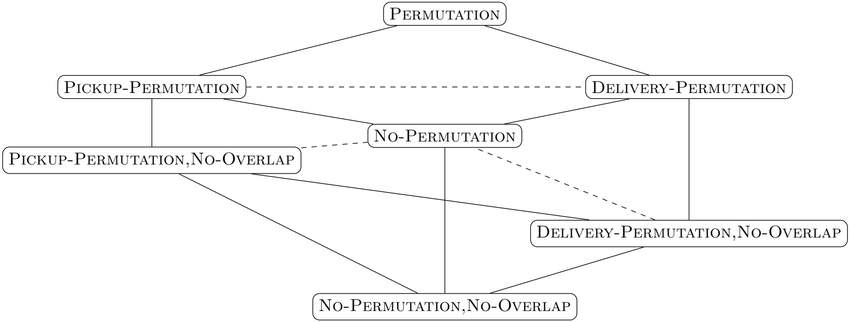
$\boldsymbol{D} = ((3, 2), (4, 5, 1))$



Variant Hierarchy



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- Let $n = |V \setminus 0|$, and choose $k_1 = n$, $k_2 = 1$ and $c^2 \equiv 0$.
- If c^1 is **metric** then **SPDP-FS** solves the **Euclidean-TSP** on $D = (G, c^1)$

No-Permutation Variants: the Splitting Subproblem

A solution to the **SPDP-FS** is **completely** described by

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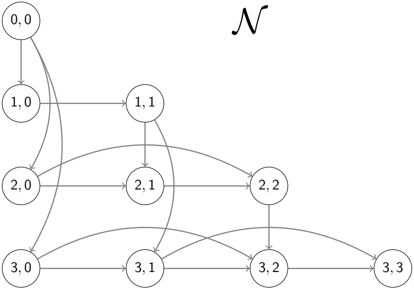
Relevance: embedding in a **2-opt heuristic** (see later)

Polynomial Algorithm for the Splitting Subproblem

No-Permutation,No-Overlap case.

Assume (wlog) $\boldsymbol{F} = (1, 2, \ldots, \boldsymbol{n})$

Approach: $(0, 0) - (\boldsymbol{n}, \boldsymbol{n})$ shortest-path in $\mathcal{N}$



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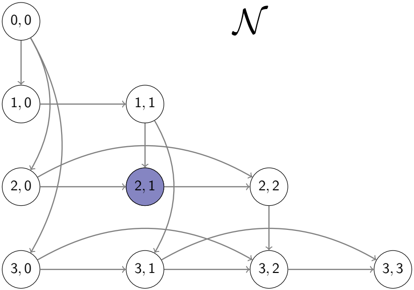

- $(\boldsymbol{i}, \boldsymbol{j})$**:** first $\boldsymbol{i}$ items picked-up and first $\boldsymbol{j}$ delivered $(\boldsymbol{i} \geq \boldsymbol{j})$

Polynomial Algorithm for the Splitting Subproblem

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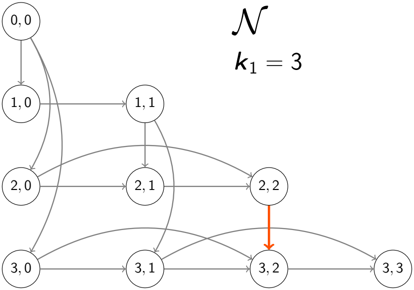

- (i, j): first i items picked-up and first j delivered ($i \geq j$)
- $(j, j) \rightarrow (i, j)$: extend P with trip $(j + 1, j + 2, \ldots, i)$

Polynomial Algorithm for the Splitting Subproblem

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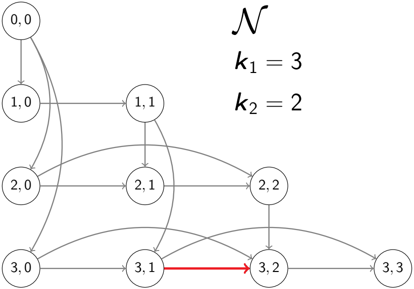

$\mathcal{N}$

$k_1 = 3$

$k_2 = 2$

- (i, j): first i items picked-up and first j delivered $(i \geq j)$
- $(j, j) \rightarrow (i, j)$: extend P with trip $(j + 1, j + 2, \ldots, i)$
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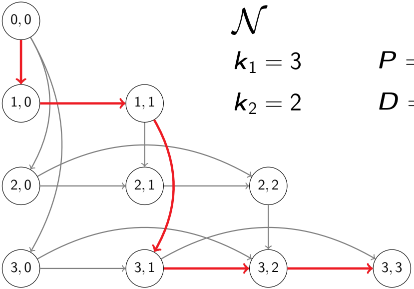

$\mathcal{N}$

$k_1 = 3 \qquad P = ((1), (2, 3))$

$k_2 = 2 \qquad D = ((1), (2), (3))$

- (i, j): first i items picked-up and first j delivered ($i \geq j$)
- $(j, j) \rightarrow (i, j)$: extend P with trip $(j + 1, j + 2, \ldots, i)$
- $(i, j_1) \rightarrow (i, j_2)$: extend D with trip $(j_1 + 1, j_1 + 2 \ldots, j_2)$
- costs preprocessed in polynomial time

Polynomial Algorithm for the Splitting Subproblem

No-Permutation,Overlap case.

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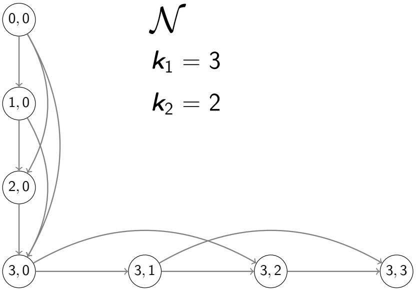
Approach: $(0, 0) - (\boldsymbol{n}, \boldsymbol{n})$ shortest-path in $\mathcal{N}$



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Polynomial Algorithm for the Splitting Subproblem

No-Permutation,Overlap case.

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$\mathcal{N}$

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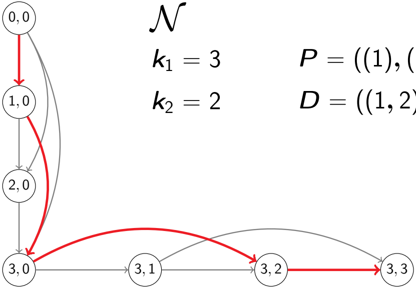

No-Permutation Variants: A 2-Opt Heuristic

No-Permutation 2-Opt Heuristic

1) F: **TSP solution** on $D = (G, c)$ with $c(e) = c^1(e) + c^2(e)$
2) (P, D): optimal splittings of F
3) Generate the **2-opt neighborhood** of F, scored by **splitting value**
4) Choose the **best neighbor** and **repeat** 3) until no improvement

Computational Results: 2-Opt Performance

Instance Set:

- **11440 instances** adapted from the Double TSP with Multiple Stacks [PM09]
- **3 classes** of 10 instances with 33, 66, 132 items respectively
  - Classes 33/66:
    $k_1 \in \{3, 6, \ldots 33/66\}$
    $k_2 \in \{3, 6, \ldots, k_1\}$
  - Class 132:
    $k_1 \in \{6, 12, \ldots, 132\}$
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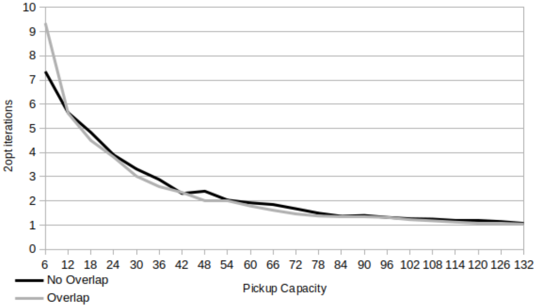
Specs:

- 1st TSP solved with CONCORDE [CON03]
- C++ compiled with `gcc 7.2 -O3`
- OS: `Linux`
- CPU: Intel `i7-3630QM` @2.40GHz



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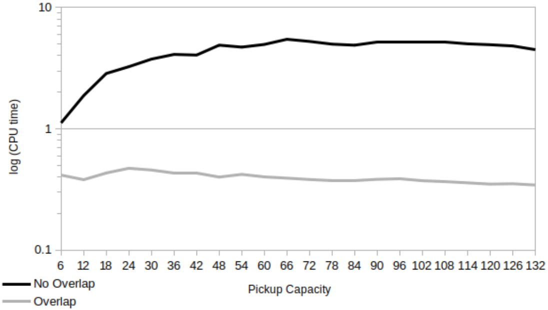

Computational Results: 2-Opt Quality

- IS: initial solution cost (1st splitting subproblem)
- FS: final solution cost (end of 2-opt heuristic)
- LB: $\mathbf{TSP}(\boldsymbol{D}^1)+\mathbf{TSP}(\boldsymbol{D}^2)$

Variant	Size	$(\mathrm{IS}-\mathrm{FS})/\mathrm{IS}$	$(\mathrm{FS}-\mathrm{LB})/\mathrm{LB}$
No-Overlap	33	0.84%	47.14%
	66	0.61%	54.27%
	132	0.41%	59.65%
Overlap	33	0.76%	45.09%
	66	0.52%	53.00%
	132	0.33%	59.07%

Table: Quality of heuristics and bounds. Results in average on all instances of the reported classes.

Conclusions and Perspectives

- **8 variants** of the SPDP-FS **formally characterized**
  - **Work in progress:** model the variants as MILPs
  - **Open:** consider other objectives (*e.g.*, completion time)
- Preliminary **complexity results** with fixed and non-fixed capacities
  - **Open:** extend the results to other capacity values and other variants

References I

[CON03] CONCORDE. D. L. Applegate, R. E. Bixby, V. Chvatal and W. J. Cook, 2003. http://www.math.uwaterloo.ca/tsp/concorde.html .

[PM09] Hanne L Petersen and Oli BG Madsen. The double travelling salesman problem with multiple stacks–formulation and heuristic solution approaches. *European Journal of Operational Research*, 198(1):139–147, 2009.

Appendix — Fixed Capacity Complexity

Proposition. The **SPDP-FS** variants

- **Permutation,No-Overlap**
- **No-Permutation,No-Overlap**

are solvable in **polynomial time** if $\boldsymbol{k}_1, \boldsymbol{k}_2 \in \{1, 2\}$.

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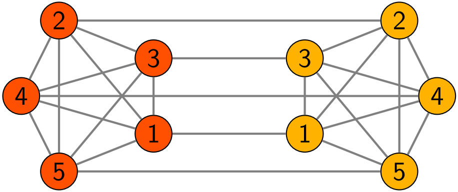
Proof.

- 2 copies v', v'' for all $v \in V \setminus 0$



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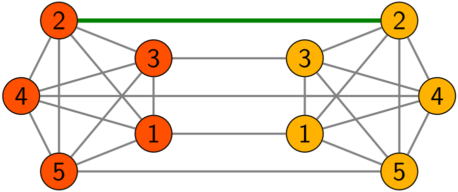
- edge (v', v'')=**trips** to collect and delivery v
- $c[v', v'']$ =cost to **collect and deliver** v



$P = \ldots(2)\ldots$
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- edge $(\boldsymbol{v}', \boldsymbol{w}')$=**best trips** to collect and deliver $\boldsymbol{v}, \boldsymbol{w}$
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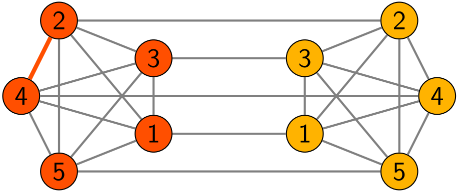

$\boldsymbol{P} = \ldots (2, 4) \ldots$

$\boldsymbol{D} = \ldots (4, 2) \ldots$

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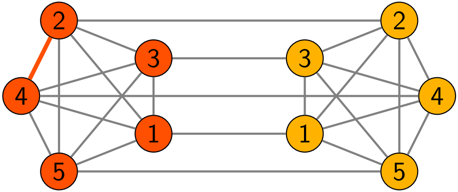

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- edge (v', w')=**best trips** to collect and deliver v, w
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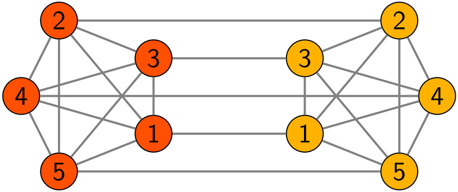
- $c[v'', w''] = 0$



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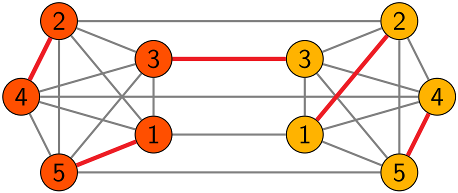

Solution=**Perfect Matching**

$P = ((2, 4), (3), (1, 5))$

$D = ((4), (2), (3), (5, 1))$

Appendix — Permutation Variants Definitions

Definition

Let $\boldsymbol{T} = (\boldsymbol{t}_1, \boldsymbol{t}_2, \ldots, \boldsymbol{t}_{\boldsymbol{k}})$ be a sequence of **trips**

We write $\boldsymbol{v} \prec_{\boldsymbol{T}} \boldsymbol{w}$ whenever $\boldsymbol{v} \in \boldsymbol{t}_{\boldsymbol{i}}$ and $\boldsymbol{w} \in \boldsymbol{t}_{\boldsymbol{j}}$ for some $1 \leq \boldsymbol{i} < \boldsymbol{j} \leq \boldsymbol{k}$

Otherwise we write $\boldsymbol{v} \not\prec_{\boldsymbol{T}} \boldsymbol{w}$

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Example

$\boldsymbol{T} = ((1, 3, 5), (4, 2))$. Then:

- $1 \prec_{\boldsymbol{T}} 4$
- $1 \not\prec_{\boldsymbol{T}} 3$
- $2 \not\prec_{\boldsymbol{T}} 5$

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Otherwise we write $\boldsymbol{v} \not\prec_{\boldsymbol{T}} \boldsymbol{w}$

PERMUTATION. The pair (P, D) is a feasible solution if and only for every $v, w \in V \setminus \{0\}$ such that $v \prec_P w$ it also holds $w \not\prec_D v$.

DELIVERY PERMUTATION. The pair (P, D) is a feasible solution if and only if:

- for every $j = 1, 2, \ldots, m$, $V(d_j)$ is a set of elements which are consecutive in the P-sequence;
- for every $v, w \in V \setminus \{0\}$, if $v \prec_D w$ then v precedes w in the P-sequence.

PICKUP PERMUTATION. The pair (P, D) is a feasible solution for every $v, w \in V \setminus \{0\}$ such that $v \prec_P w$ we also have that v precedes w in the D-sequence.

Appendix — Pickup vs. Delivery Permutation

Let $k_1 = k_2 = 3$ and $n = 5$.

- **Pickup Permutation OK, Delivery Permutation NOT:**
  - $P = ((1, 5, 4), (2, 3))$
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  - $F = (1, 5, 4, 3, 2)$ if pickup permutes
  - $F = (1, 5, 4, 2, 3)$ if pickup cannot permute
- **Delivery Permutation OK, Pickup Permutation NOT:**
  - $P = ((1, 5, 4), (2, 3))$
  - $D = ((1), (5, 2, 4), (3))$