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Optimal Planning of Waste Sorting Operations through Mixed Integer Linear Programming

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1

Circular Economy

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A circular economy is based on the principles of designing out waste and pollution, keeping products and materials in use, and regenerating natural systems.

“ ELLEN **MACARTHUR**

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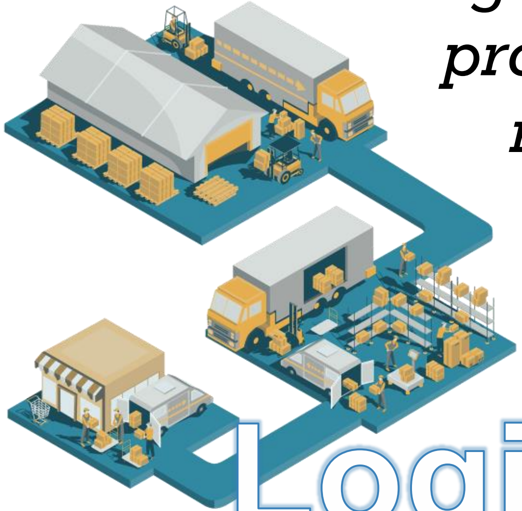

Logistics

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Process of planning, implementing and controlling the cost-effective flow of raw materials, in-process inventory, finished goods and related information…

Logistics

- **Logistics (Forward)**

…from the point of origin to the point of consumption for the purpose of conforming to customer requirement

- Council of Logistics, 1988 -

- **Reverse Logistics**

…from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal

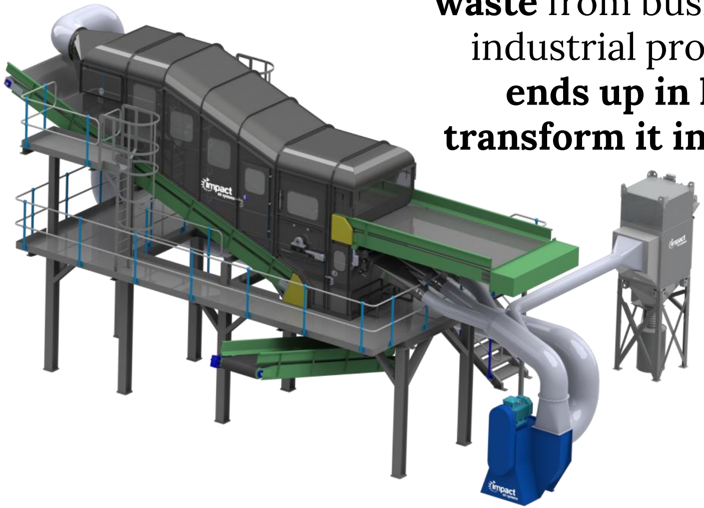
- Rogers and Tibben-Lembke -

Waste
◉ Collection

Waste
◉ Sorting

Waste collection is the transfer of solid waste from the point of use and disposal to the point of treatment. **Industrial waste** refers to solid waste **generated by manufacturing or industrial processes**.

Waste sorting is the process by which the waste is separated into different elements (paper, plastics, glass,...) . **The aim is to minimize the amount of waste** from businesses and industrial processes **that ends up in landfill and transform it into valuable resources.**



Reverse logistics – main issues

Low margin business

Profit margins of waste managment companies are extremely low. Abscence of pricing levers and high percentage of operation and logistics costs

Uncertainties in supplies

Waste quality matters! Each waste container differs in its materials composition, hence in its extractable value. This value is not known before the sorting process is completed

Flows tracking difficulties

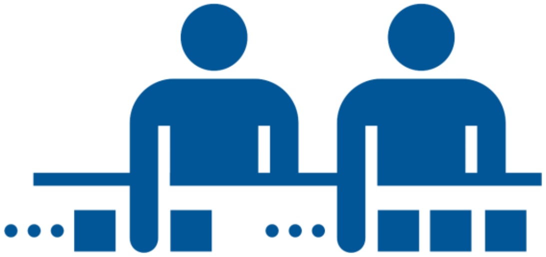
Materials flows are not easiy measurable, both in terms of weight or volume. Data quality activities and related investments must be considered for developing relying support systems.

2 Sorting operations Planning

Production Planning – Lot sizing models review

Lot sizing model extensions are influenced by several topics regarding set ups, production, inventory and demand scenarios*

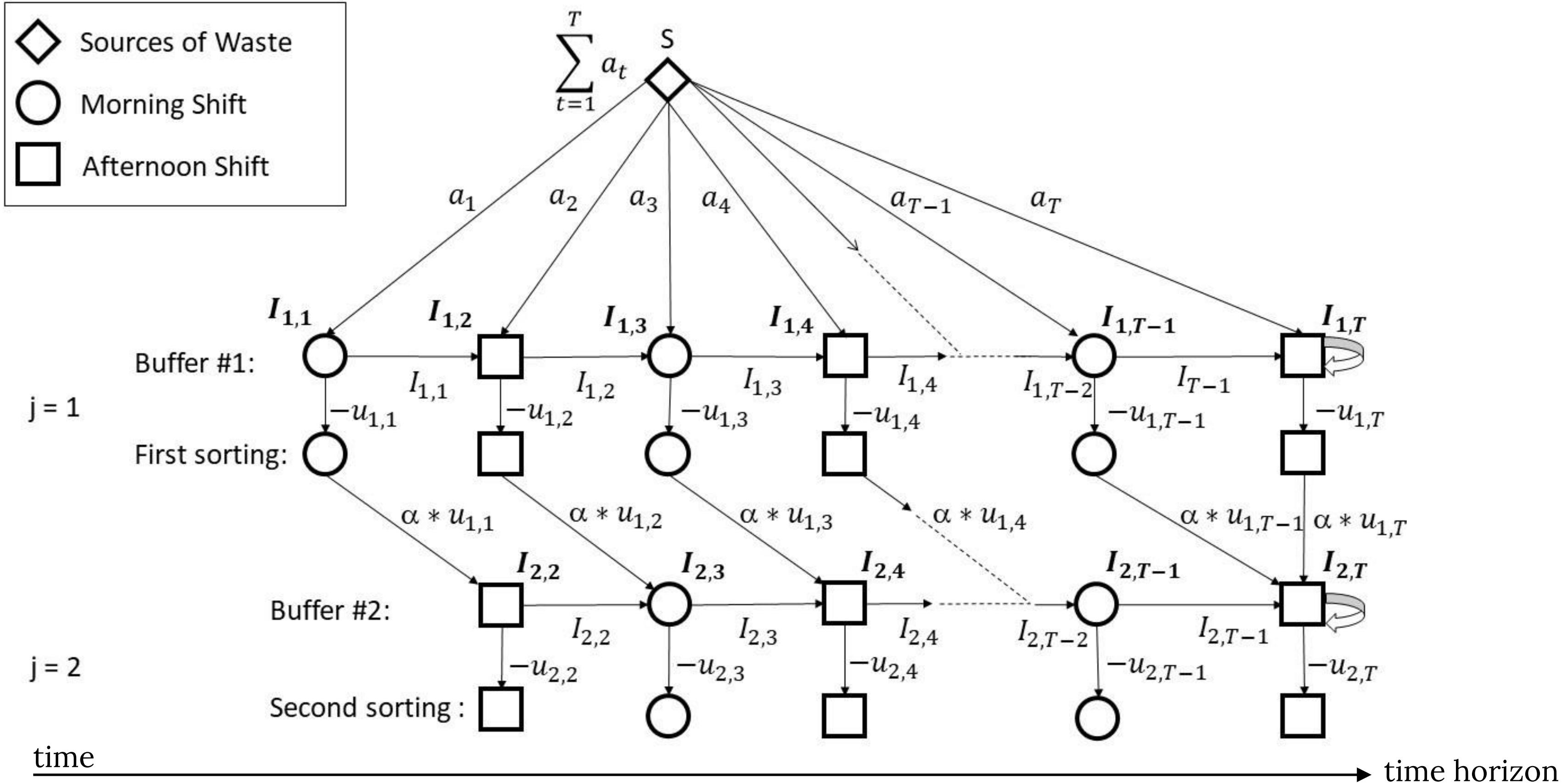
The numerous extensions of the basic lot sizing problem show that it can be used to model a variety of industrial problems*



We believe boundaries between lot sizing and scheduling are fading and further integration of lot sizing, sequencing, loading and workforce allocation constitute a challenging research track

*Raf Jans, Zeger Degraeve. Modeling Industrial Lot Sizing Problems: A Review. International Journal of Production Research, Taylor & Francis, 2008, 46 (06), pp.1619-1643.

A **mixed integer linear program** is used to schedule the selection operations of a two-phase waste selection process



MILP - parameters

$j = \{1,\ldots,J\}$: index of the J sorting stages
$p = \{1,\ldots,P\}$: index of the P time-shifts
T : time horizon partitioned in time shifts with $t \in \{1,\ldots,T\} = T_1 \cup \ldots \cup T_P$
C : hourly cost of each operator
σ_t : working hours for time t determined by the corresponding shift p
$C_t = C * \sigma_t$: cost of each operator at time t
f_j : set-up cost of sorting stage j
a_t : quantity of material in kg unloaded from trucks at time t
α_j : percentage of waste processed in stage $j-1$, received in input by buffer j
S_j : maximum inventory capacity of the sorting stage buffer j
LC_j : critical stock level threshold of buffer j
ρ_j : fraction of material allowed to be left at buffer j at the end of time horizon
K_j : single operator hourly production capacity [kg/h] of sorting stage j
$SK_{j,t} = K_j * \sigma_t$: operator sorting capacity in sorting stage j, at time t
M : maximum number of operators available in each time shift
E_j : minimum number of operators to be employed in each time shift of stage j
∂h^i_j : slope of the i-th part of linearization of the buffer j stock cost curve

MILP - parameters - Time partioned in working shift

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MILP - parameters - Time dimension

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MILP - parameters - Stock costs

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MILP - variables

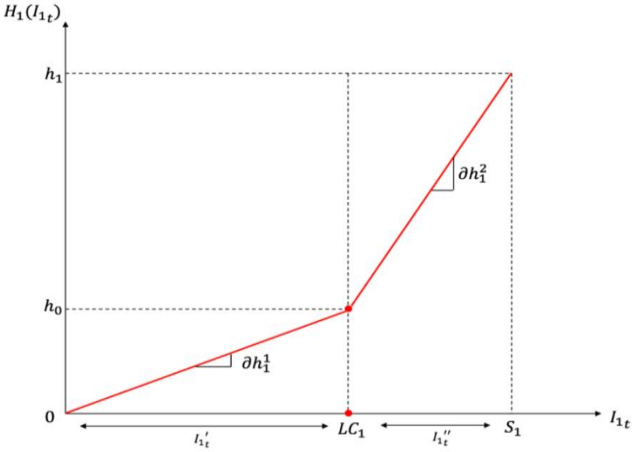
$x_{j,t} \in \mathbb{Z}^+$: operators employed in the sorting stage j at time t

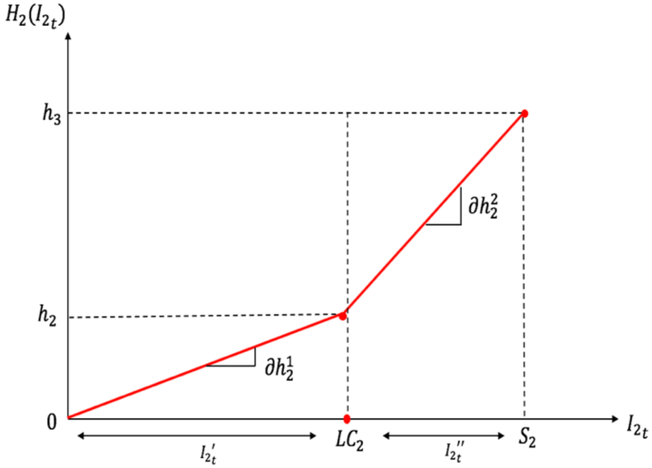
$u_{j,t} \in \mathbb{R}^+$: processed quantity at stage j at time t

$y_{j,t} \in \{0,1\}$: equal to 1 if stage j is activated at time t , 0 otherwise

$I_{j,t} = I'_{j,t} + I''_{j,t} \geq 0$: stock level of material in buffer j at time t; for each stage j the corresponding $I'_{j,t}$ and $I''_{j,t}$ represent the inventory level before and after reaching the critical threshold respectively.

$w_{j,t} \in \{0,1\}$: equal to 1 if $I''_{j,t} > 0$, 0 otherwise. Indeed, this binary variables are used to model the piece-wise linear functions of the buffer stock costs.





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MILP - features

- Work shifts **scheduling**

$y_{j,t} \in \{0,1\}$: equal to 1 if stage j is activated at time t, 0 otherwise

- Production **lot-sizing**

$u_{j,t} \in \mathbb{R}^+$: processed quantity during stage j at time t

- **Workforce allocation**

$x_{j,t} \in \mathbb{Z}^+$: operators employed in sorting stage j at time t

$$\min Z = \sum_{j\in J}\sum_{t\in T} C_t x_{j,t} + \sum_{j\in J}\sum_{t\in T} f_j y_{j,t} + \sum_{j\in J}\sum_{t\in T}\left(\partial h_j^1 I'_{j,t} + \partial h_j^2 I''_{j,t}\right) \tag{1}$$

s.t.

$$E_j\, y_{j,t} \le x_{j,t} \le M\, y_{j,t} \qquad \forall j \in J, t \in T_p, p \in P \tag{2}$$

$$\sum_{j\in J} x_{j,t} \le M \qquad \forall t \in T \tag{3}$$

$$u_{j,t} \le SK_{j,t}\, x_{j,t} \qquad \forall j \in J, t \in T \tag{4}$$

$$I_{1,t} = I_{1,t-1} + a_t - u_{1,t} \qquad \forall t \in T \setminus 0 \tag{5}$$

$$I_{j,t} = I_{j,t-1} - u_{j,t} + \alpha_j\, u_{j-1,t} \qquad \forall t \in T \setminus 0, j \in J \setminus 1 \tag{6}$$

$$I_{j,t} = I'_{j,t} + I''_{j,t} \qquad \forall j \in J, t \in T \tag{7}$$

$$LC_j\, w_{j,t} \le I'_{j,t} \le LC_j \qquad \forall j \in J, t \in T \tag{8}$$

$$0 \le I''_{j,t} \le (S_j - LC_j)\, w_{j,t} \qquad \forall j \in J, t \in T \tag{9}$$

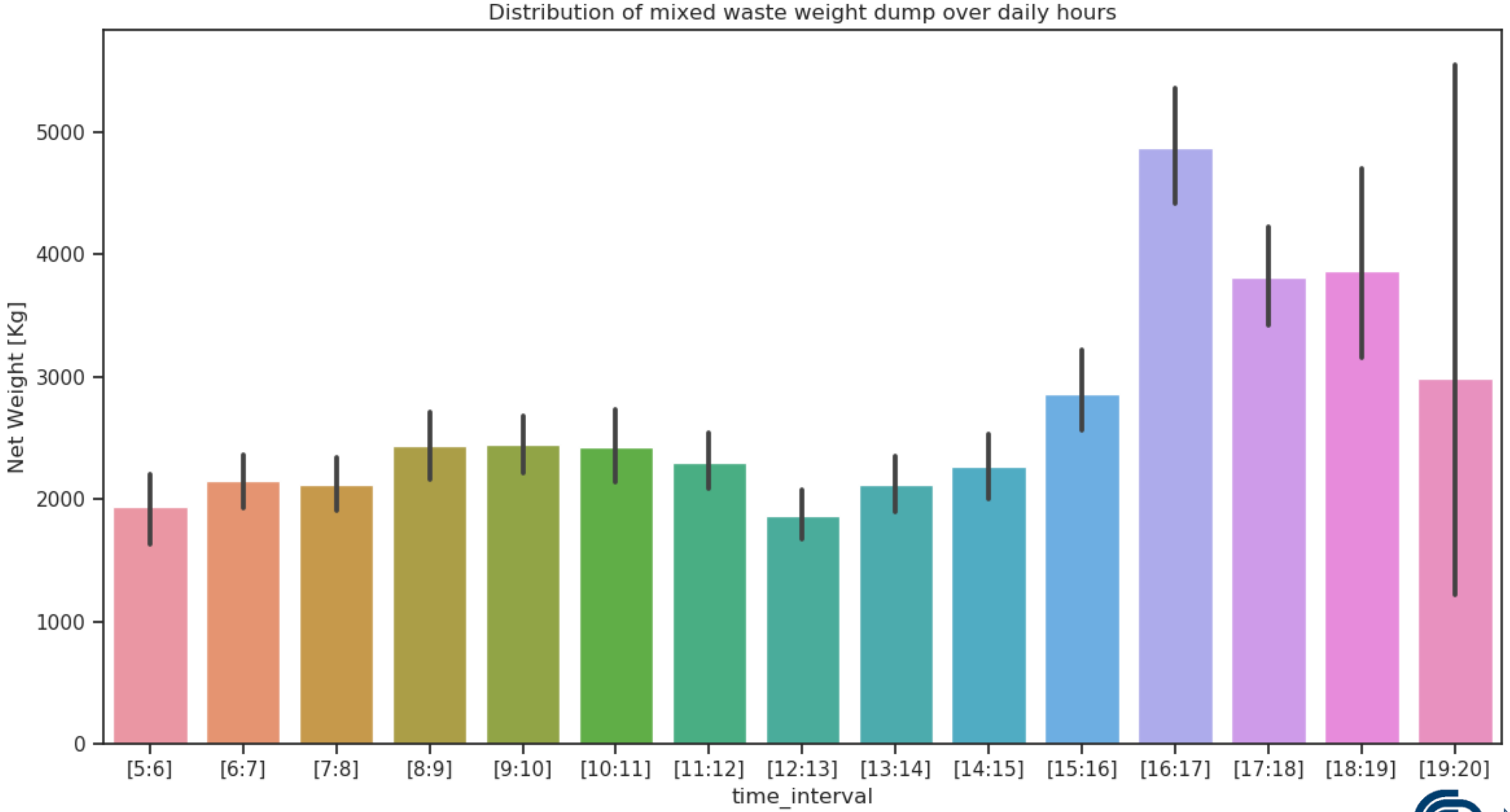
$$I_{j,T} \le \rho_j\, LC_j \qquad \forall j \in J \tag{10}$$

$$x_{j,t} \in \mathbb{Z}^+ \qquad \forall j \in J, t \in T \tag{11}$$

$$u_{j,t} \in \mathbb{R}^+ \qquad \forall j \in J, t \in T \tag{12}$$

$$y_{j,t} \in \{0,1\} \qquad \forall j \in J, t \in T \tag{13}$$

Daily distribution of waste arrivals



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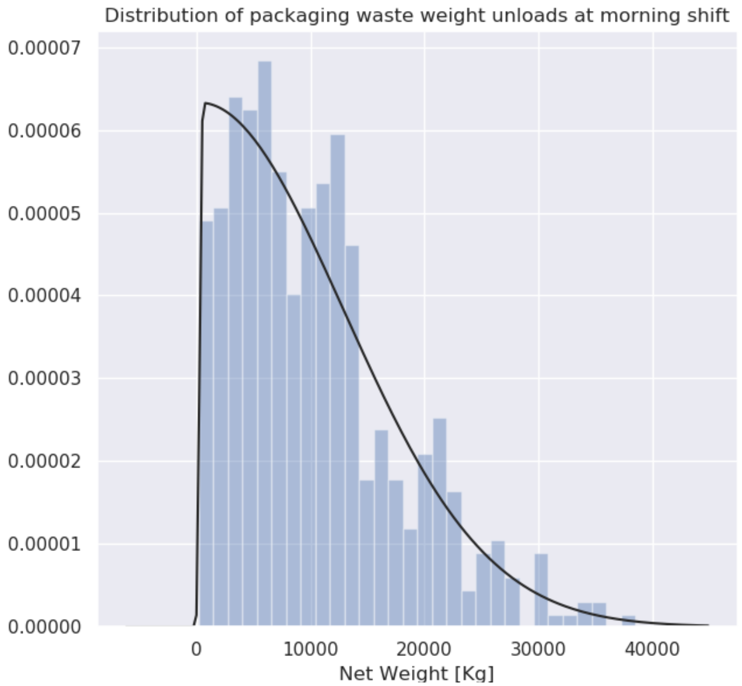
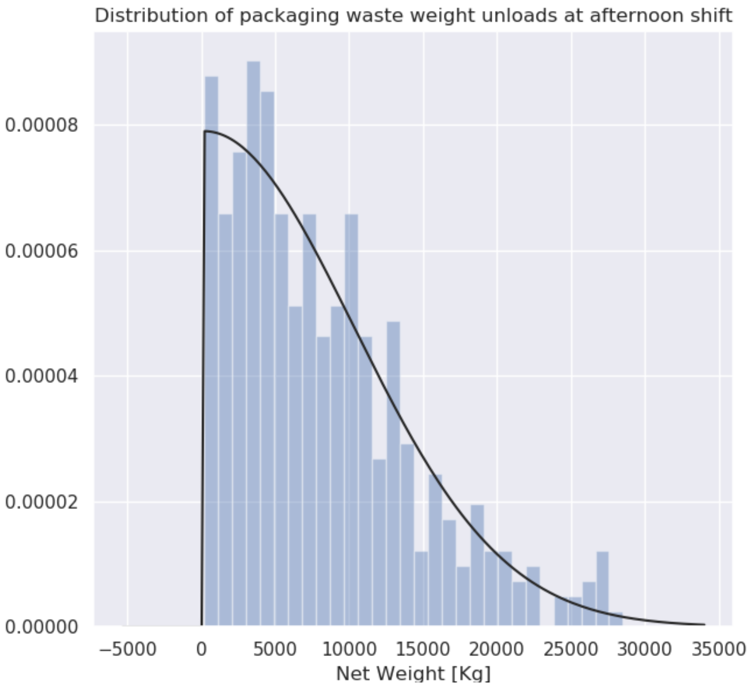
MILP – instances creation

- Incoming weight dimension

Arrivals	Shift_1 LB	Shift_1 UB	Shift_2 LB	Shift_2 UB
low	200	3250	200	2600
moderate	3260	6600	2600	5400
average	6600	10800	5400	8650
high	10800	16500	8660	13400
extreme	16500	38000	13400	28000

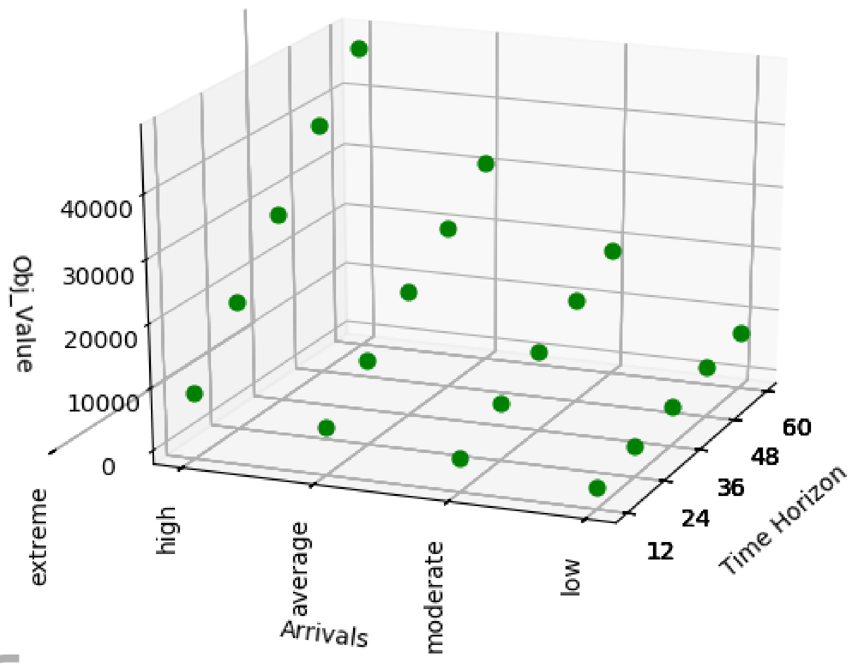
- Time horizon dimension: $T \in \{12,24,36,48,60\}$



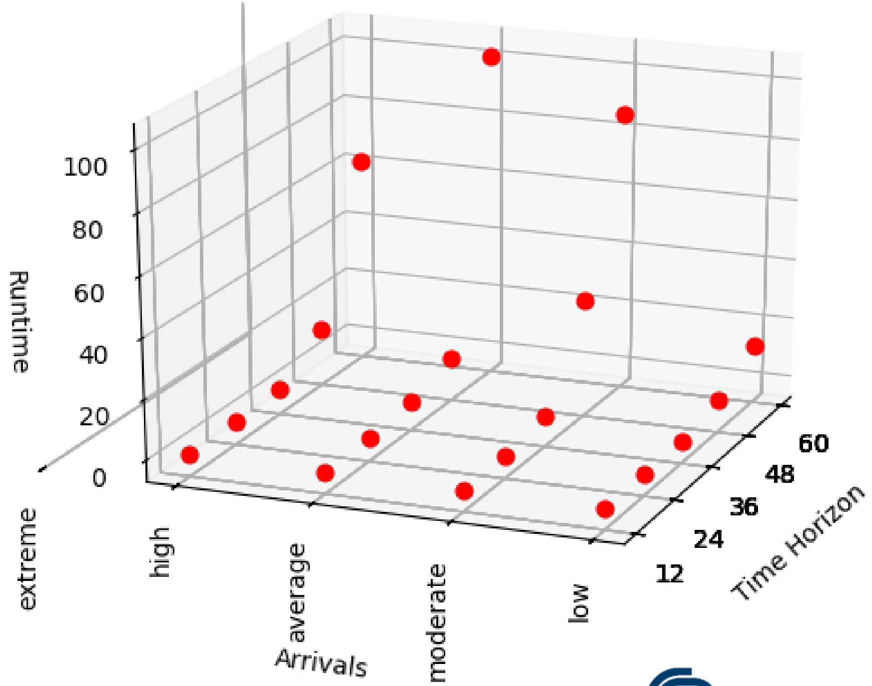


MILP – model testing on 5x5 instances grid

- Optimal objective value

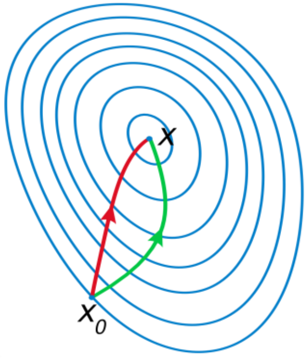


- Runtime



MILP – Results #1

Comparison of optimal **objective value** and **runtime** between the presented **MILP** and the actual waste **company solution**



problems are exactly solved via branch-and-bound using *Gurobi* 8 solver

Instances TH - Arrivals	ObjVal_M	Obj_Val_C	%Obj reduction	Runtime_M	Runtime_C	%Run diff
12 - low	1151.036	1263.556	8.90	0.036	0.029	-0.241
12 - moderate	3233.546	3421.400	5.50	0.052	0.033	-0.576
12 - average	5388.570	6319.804	14.7	0.092	0.081	-0.136
12 - high	8262.336	9117.556	9.4	0.271	0.050	-4.420
12 - extreme	infeasible	infeasible	infeasible	infeasible	infeasible	infeasible
24 - low	2708.048	3094.936	12.5	0.131	0.058	-1.259
24 - moderate	6933.418	7659.156	9.5	0.446	0.126	-2.540
24 - average	11502.106	13507.594	14.8	0.477	0.748	0.362
24 - high	18320.334	20186.248	9.2	2.217	0.155	-13.303
24 - extreme	infeasible	infeasible	infeasible	infeasible	infeasible	infeasible
36 - low	4104.140	4564.886	10.1	0.333	0.152	-1.191
36 - moderate	10577.530	11834.330	10.6	7.372	1.154	-5.388
36 - average	17902.680	20485.472	12.6	6.530	31.161	0.790
36 - high	28025.228	30655.698	8.6	1.837	0.361	-4.089
36 - extreme	infeasible	infeasible	infeasible	infeasible	infeasible	infeasible
48 - low	5952.454	7030.596	15.3	6.667	2.150	-2.101
48 - moderate	14472.414	15962.360	9.3	64.374	6.642	-8.692
48 - average	23870.748	26794.730	10.9	12.559	29.889	0.580
48 - high	38239.526	41147.530	7.1	20.576	2.148	-8.579
48 - extreme	infeasible	infeasible	infeasible	infeasible	infeasible	infeasible
60 - low	7105.006	8443.570	**15.9**	26.816	2.697	-8.943
60 - moderate	18328.174	20426.798	10.3	140.476	62.179	-1.259
60 - average	30355.192	34709.392	12.5	148.196	300.026	0.506
60 - high	47101.504	50434.394	6.6	98.605	1.611	-60.207
60 - extreme	infeasible	infeasible	infeasible	infeasible	infeasible	infeasible

MILP – Results #2

**Gap termination tolerance is set to 0.25% ad time limit to 1500s*

We attempt to solve instances with a greater number of J sorting stages, up to 5 subsequent stages. These instances also differ in time horizon but share the same kind of average magnitude of arriving waste quantity

Instances Settings		Status	Gap	Obj_Val	RunTime	NodeCount	NumConstr	NumVars	NumBinVar	NumIntVar
TH = 36,	J = 2	optimal*	0	20608	5.59	30241	542	504	144	216
TH = 36,	J = 3	optimal*	0	22500	89.68	196932	795	756	216	324
TH = 36,	J = 4	optimal*	0	23472	353.64	572552	1048	1008	288	432
TH = 36,	J = 5	time limit	0.0109	23982	1500.00	755240	1301	1260	360	540
TH = 48,	J = 2	optimal*	0	28283	36.67	102454	722	672	192	288
TH = 48,	J = 3	time limit	0.0072	29771	1500.00	2328238	1059	1008	288	432
TH = 48,	J = 4	time limit	0.0139	31155	1500.00	849918	1396	1344	384	576
TH = 48,	J = 5	time limit	0.0314	33026	1500.00	382850	1733	1680	480	720
TH = 60,	J = 2	optimal*	0	34231	301.06	764941	902	840	240	360
TH = 60,	J = 3	time limit	0.0144	38793	1500.00	1489421	1323	1260	360	540
TH = 60,	J = 4	time limit	0.0241	40767	1500.00	669998	1744	1680	480	720
TH = 60,	J = 5	time limit	0.0343	39889	1500.00	337922	2165	2100	600	900

MILP – Results #3

**Gap termination tolerance is set to 0.25% ad time limit to 1500s*

Formulation costs tuning can be suitable whenever the decision maker wants to foster a scattered production schedule with a bigger lot size by removing stock costs, or to encourage a lean production by removing the production costs

		Status	Gap	Obj_Val	RunTime	NodeCount	NumConstr	NumVars	NumBinVar	NumIntVar
TH = 36, J = 2,	Prod: v, Stock: v	optimal*	0	20099	4.22	19205	542	504	144	216
TH = 36, J = 2,	Prod: v, Stock: x	optimal*	0	18605	4.56	11326675	326	288	72	144
TH = 36, J = 2,	Prod: x, Stock: v	optimal*	0	123	0.98	1	542	504	144	216
TH = 48, J = 2,	Prod: v, Stock: v	optimal*	0	27856	118.43	373014	722	672	192	288
TH = 48, J = 2,	Prod: v, Stock: x	optimal*	0	25750	6.33	2188403	434	384	96	192
TH = 48, J = 2,	Prod: x, Stock: v	optimal*	0	228	0.18	1	722	672	192	288
TH = 60, J = 2,	Prod: v, Stock: v	optimal*	0	33845	585.24	1491897	902	840	240	360
TH = 60, J = 2,	Prod: v, Stock: x	optimal*	0	31200	9.38	2196713	542	480	120	240
TH = 60, J = 2,	Prod: x, Stock: v	optimal*	0	217	0.21	1	902	840	240	360

MILP – WebApp development

REMIND - Reverse Manufacturing INnovation Decision system

- Home
- Admin
- Waste Selection Planner
- Route Planner
- Profitability (on work)

Seleziona il periodo di cui vuoi conoscere la programmazione ottimale delle attività di selezione

Inizio programmazione: mm/dd/yyyy

Fine programmazione : mm/dd/yyyy

Cerca

Soluzione ottima:

Data	Turno	1a selezione	2a selezione	operatori 1a sel.	operatori 2a sel.	Quantità 1a sel.	Quantità 2a sel.
21/09/2020	mattina	attivata	-	6	0	9000	0
21/09/2020	pomeriggio	attivata	-	8	0	6240	0
22/09/2020	mattina	attivata	-	8	0	12480	0
22/09/2020	pomeriggio	attivata	-	8	0	6239	0
23/09/2020	mattina	attivata	-	8	0	12480	0
23/09/2020	pomeriggio	attivata	-	8	0	6240	0
24/09/2020	mattina	attivata	-	8	0	12480	0
24/09/2020	pomeriggio	-	attivata	0	8	0	5280
25/09/2020	mattina	attivata	-	8	0	12479	0
25/09/2020	pomeriggio	attivata	-	8	0	6240	0
26/09/2020	mattina	attivata	-	8	0	12480	0
26/09/2020	pomeriggio	-	attivata	0	7	0	4174

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Current and future works

- Introducing **robustness** to supplies uncertainties making use of probabilistic bounds of constraints violations
- Considering the **fractions of materials** moving to the following sorting phase within the sorting operations planning model
- Develop a production model version that includes **waste baling** press operations
- Considering **production capacity as a function of operators employed**
- **Avoid linearizing** the quadratic curves of stock costs