Additive Tree $O(\rho \log n)$-Spanners from Tree Breadth ρ

Dieter Rautenbach

Universität Ulm

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Joint with Oliver Bendele

Spanners

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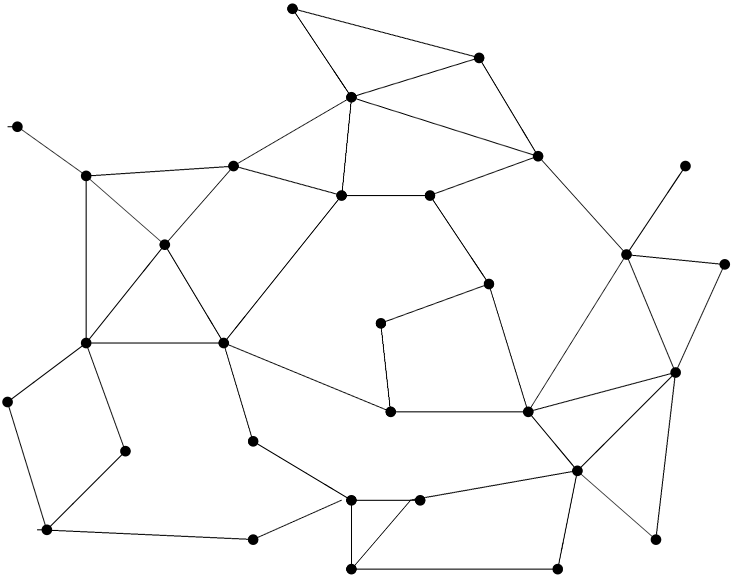

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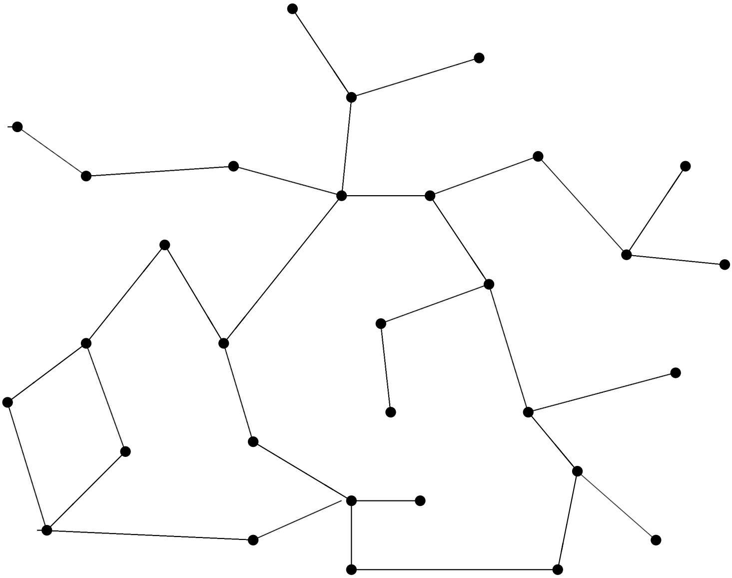

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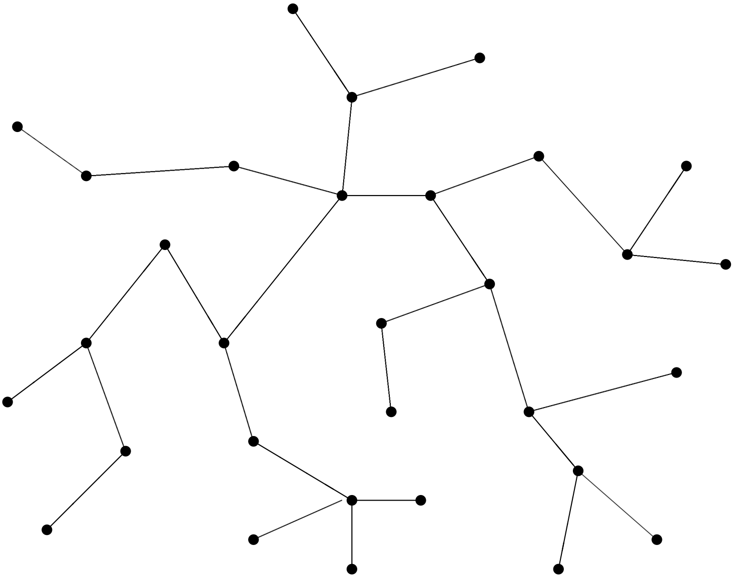

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Replacing
$$\text{“}d_H(u, v) \leq d_G(u, v) + k\text{”}$$
with
$$\text{“}d_H(u, v) \leq k \cdot d_G(u, v)\text{”}$$
yields the multiplicative versions.

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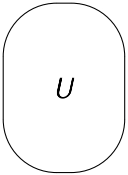

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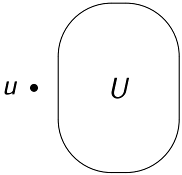

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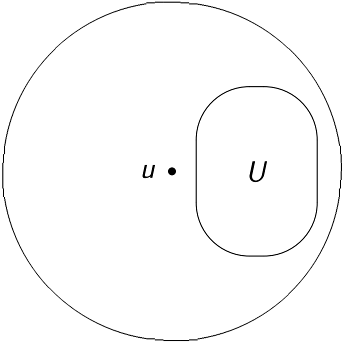

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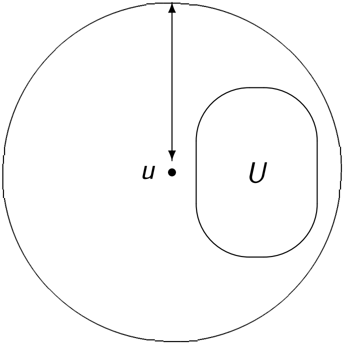

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  Tree breadth is NP-hard.
- (Dourisboure and Gavoille 2007, for tree length)
  A tree decomposition of breadth at most $6\mathrm{tb}(G) + 1$ can be found in linear time.

Tree breadth $\Rightarrow$ spanners

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Let G be a connected graph of order n, size m, and tree breadth ρ.

Theorem (Dragan and Köhler 2014)

Given G as above, one can construct in time $O(m \log n)$ a multiplicative tree $O(\rho \log n)$-spanner of G.

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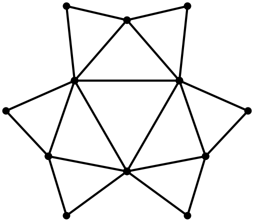

Figure: The graph G_3.

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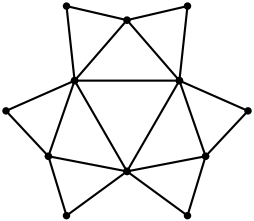

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(Kratsch, Le, Müller, Prisner, and Wagner 2002)

G_k admits no additive tree $\mathrm{tb}(G_k) \log_2 \left(\frac{n(G_k)}{3}\right)$-spanner

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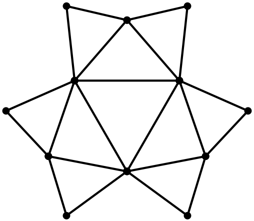

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For a tree T, let $\mathrm{pbt}(T)$ be the maximum depth of a perfect binary tree that is a topological minor of T.

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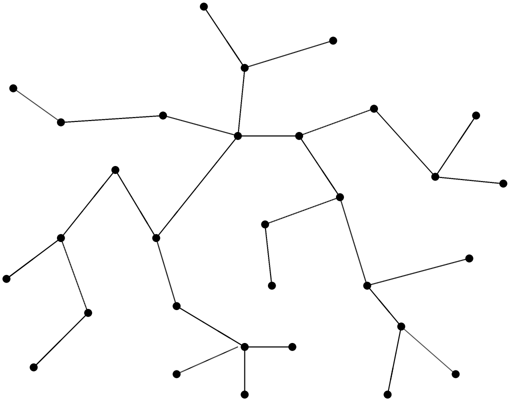

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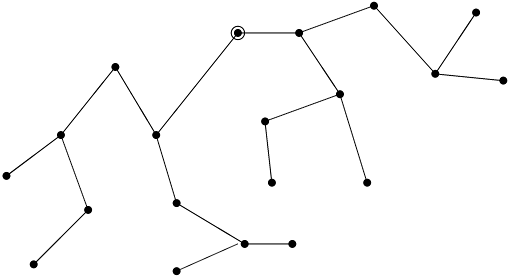

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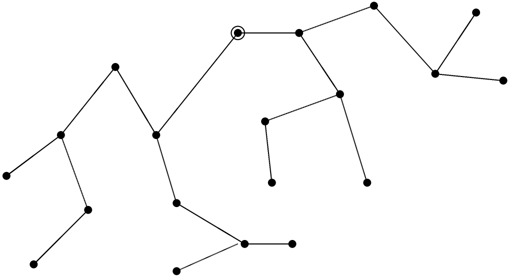

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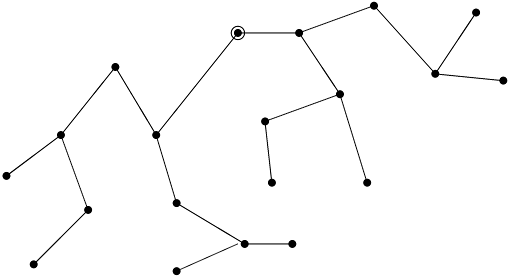

$\Rightarrow 3 \leq \mathrm{pbt}(T) \leq O(\log(n(T)))$.

Tree breadth $\Rightarrow$ spanners

Theorem (Bendele and R 2020)

Given G and tree decomposition $\big(T, (X_t)_{t \in V(T)}\big)$ of G of breadth ρ, one can construct in time $O\big(m \cdot \mathrm{pbt}(T)\big)$ an

additive tree $O\big(\rho \cdot \mathrm{pbt}(T)\big)$-spanner of G.

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Corollary (Bendele and R 2020)

Given G and given a

multiplicative tree k-spanner T of G,

one can construct in time $O\big(mn\big)$ an

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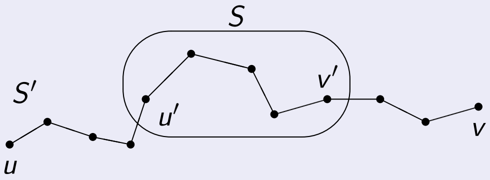

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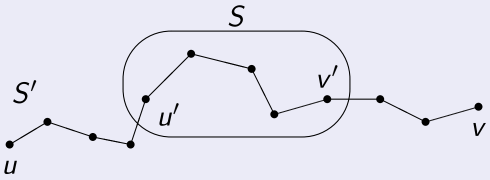

$d_{S'}(u, v)$

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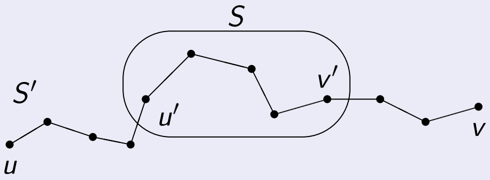

$$d_{S'}(u, v) \quad = \quad d_{S'}(u, u') + d_S(u', v') + d_{S'}(v', v)$$

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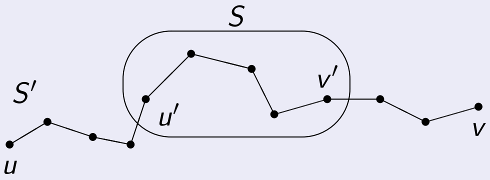

$$\begin{aligned} d_{S'}(u,v) &= d_{S'}(u,u') + d_S(u',v') + d_{S'}(v',v) \\ &\leq \rho' \end{aligned}$$

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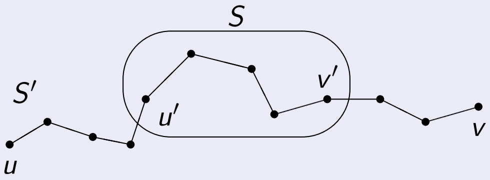

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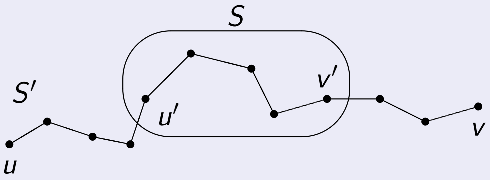

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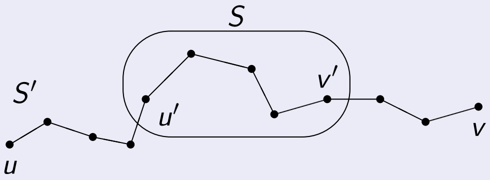

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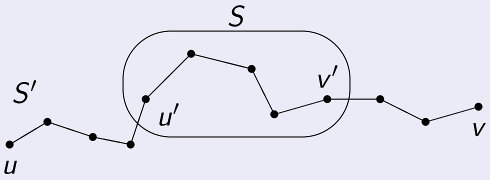

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d_{S'}(u,v) &= d_{S'}(u,u') + d_S(u',v') + d_{S'}(v',v) \\
&\leq \rho' + (d_G(u',v') + \rho) + \rho' \\
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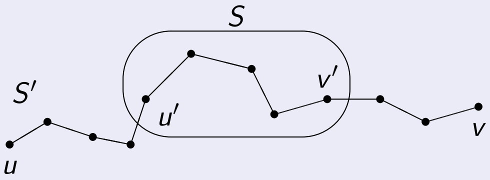

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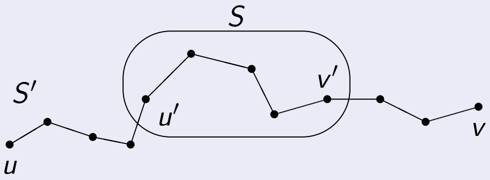
d_{S'}(u, v) &= d_{S'}(u, u') + d_S(u', v') + d_{S'}(v', v) \\
&\leq \rho' + (d_G(u', v') + \rho) + \rho' \\
&\leq \rho + 2\rho' + d_G(u', u) + d_G(u, v) + d_G(v, v') \\
&\leq \rho + 2\rho'
\end{aligned}
$$

Proofs

Proof.



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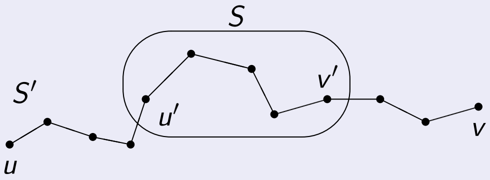

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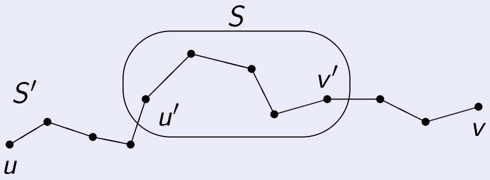

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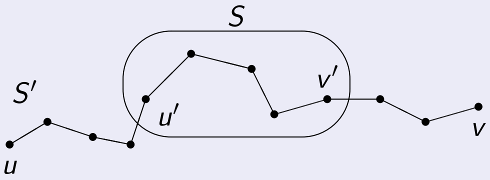

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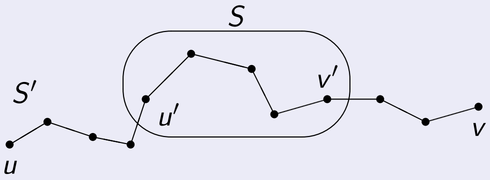

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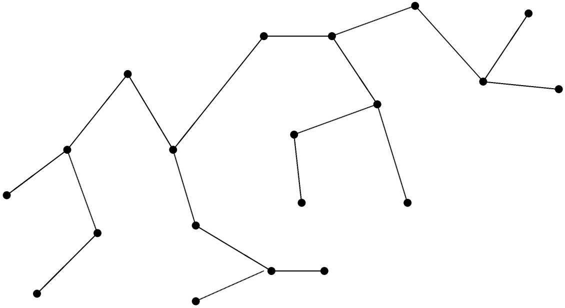
Let T be a tree.



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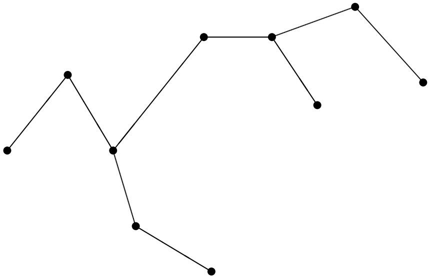

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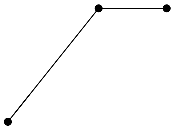

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T_0

Proofs

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$T_0 \supset T_1$

Proofs

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$$T_0 \supset T_1 \supset T_2$$

Proofs

Let T be a tree.

- $$T_0 \supset T_1 \supset T_2 \supset \ldots \supset T_{d(T)},$$

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Lemma (Bendele and R 2020)

$\mathrm{pbt}(T) = d(T)$ *for every tree* T.

Proofs

Lemma (Bendele and R 2020)

Given G and a tree decomposition $\left(T, (X_t)_{t \in V(T)}\right)$ of breadth ρ, one can construct in time $O(m \cdot \mathrm{d}(T))$ a $16\rho\mathrm{d}(T)$-additive subtree S of G

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Thank you for the attention!