A multiperiod drayage problem with flexible service periods

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Outline

- A new drayage problem;
- Three different approaches for this problem:
  - a path-based formulation (PBF) with all feasible routes (by an off-the-shelf MIP solver);
  - a Price-and-Branch algorithm ($P\&B$);
  - a reformulation by node-arc model ($SEAF$) by the MIP solver;
- Analysis of their effectiveness;
- Changes in flexibility & costs for the carrier and customers;

The problem

- *Drayage*: last-mile transportation of containers by trucks from an intermodal facility (e.g. a port) to serve customers;
- A homogeneous fleet of 20ft containers;
- Two types of customers:
  - *Importers* receiving container loads from the port;
  - *Exporters* shipping container loads to the same port;
- A fleet of one-container and two-containers trucks;
- Drivers waiting for containers in the facilities of customers;
- Empty containers as well as loaded containers;

The problem

- Two types of customers in terms of flexibility:
  - *Inflexible* customers, who require to be served only at the due day;
  - *Flexible* customers, for whom the carrier pays customer-dependent penalties for earlier/later than desired services within customer-dependent periods;
- When should they be served? Which routes should be made to minimize routing costs?
- A maximum number of container loads can be delivered or collected early or late in each period for *flexible* customers;
- A capacity and a cost for containers left at the port for late delivery and early collection.

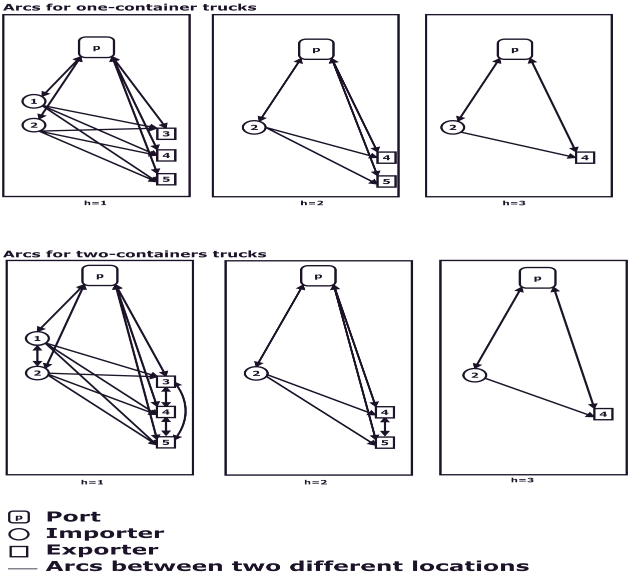
Literature

- Many papers on single-day drayage problems (Ghezelsoflu et al., 2018);
  - Few papers with two-container trucks;
  - Many papers on the delivery or collection of containers (instead of container loads);
  - Many heuristics;
- Several routing problems with multiple periods:
  - Inventory routing problems :-(
  - Periodic routing problems :-(
  - Flexible Vehicle routing problem :-|
  - Multiperiod Vehicle Routing Problems :-)

1 - Path-based formulation with all feasible routes

- H: set of periods in the planning horizon;
- $G = (N, A)$: *physical* directed graph;
- $N = \{p\} \cup V = \{p\} \cup I \cup E$, i.e., nodes are the port (p) and all possible customers $V = I \cup E$, in which I and E are the set of importers and exporters, respectively;
- $(i, j) \in A$: the direct truck trip between i and j, with two associated costs c_{ij}^1 and c_{ij}^2 for one- and two-containers trucks, respectively;
- (Physical) sub-graphs $G_h^t = (N_h, A_h^t)$ of G for each period $h \in H$ and truck type $t \in T = \{1, 2\}$;
- $N_h = \{p\} \cup V_h$, $V_h = I_h \cup E_h$ and I_h and E_h set of importers and exporters accepting a transportation service in period $h \in H$, respectively; A_h^t: arcs induced by N_h and feasible for the given truck type $t \in T$;

1 - Path-based formulation with all feasible routes



1 - Path-based formulation with all feasible routes

- $R(h)^t$: set of all feasible routes for trucks of type t in period h (i.e. cycles in G_h^t starting and ending in p);
- Self-loops for two-container trucks;
- $R(h) = R(h)^1 \cup R(h)^2$: set of all feasible routes in period $h \in H$;
- $R = \cup_{h \in H} R(h)$;
- $R(1:h) = \cup_{k=1 \ldots h} R(k)$: set of feasible routes *up to* day $h \in H$;
- k_h^t: number of trucks of type $t \in T$ available in period $h \in H$;
- For each route $r \in R(h)$, let $\alpha_{v,r} =$
  $$\begin{cases} 0 & \text{if customer } v \text{ is not visited in route } r \\ 1 & \text{if customer } v \text{ is served by 1 container in route } r \\ 2 & \text{if customer } v \text{ is served by 2 containers in route } r; \end{cases}$$
- d_v^h: the demand of customer v in period $h \in H$;
- $d_v^{1:h} = \sum_{k=1 \ldots h} d_v^k$: demand of customer $v \in V$ up to day $h \in H$;

1- Path-based formulation with all feasible routes

Decision variables:

- x_r: how many times route $r \in R$ is performed, c_r is the unitary cost;
- s_v^{h+}: number of container loads delivered later/collected earlier than agreed for customer $v \in V$ in period $h \in H$, f_v^{h+} is the unitary cost;
- s_v^{h-}: number of container loads collected later/delivered earlier than agreed for customer $v \in V$ in period $h \in H$, f_v^{h-} is the unitary cost;

The path-based formulation (PBF):

$$\min \quad \sum_{r \in R} c_r x_r + \sum_{h \in H} \sum_{v \in V} (f_v^{h+} s_v^{h+} + f_v^{h-} s_v^{h-}) \tag{1}$$

$$s.t. \quad \sum_{r \in R(1:h)} \alpha_{v,r} x_r + s_v^{h+} - s_v^{h-} = d_v^{1:h} \qquad v \in V \ , \ h \in H \tag{2}$$

$$\sum_{r \in R^t(h)} x_r \leq k_h^t \qquad t \in T \ , \ h \in H \tag{3}$$

$$s_v^{h+} \leq u_v^{h+} \qquad v \in V \ , \ h \in H \tag{4}$$

$$s_v^{h-} \leq u_v^{h-} \qquad v \in V \ , \ h \in H \tag{5}$$

$$\sum_{v \in I} s_v^{h+} - \sum_{v \in I} s_v^{h-} \leq u_h^I \qquad h \in H \tag{6}$$

$$\sum_{v \in E} s_v^{h-} - \sum_{v \in E} s_v^{h+} \leq u_h^E \qquad h \in H \tag{7}$$

$$x_r \in \mathbb{N} \qquad r \in R \tag{8}$$

$$s_v^{h+} \ , \ s_v^{h-} \in \mathbb{R}_+ \qquad v \in V \ , \ h \in H \tag{9}$$

2 - Price-and-Branch for the path-based formulation

- When the number of customers grows, the number of feasible routes quickly becomes too large;
- The linear relaxation ($\underline{PBF}$) of (PBF) can be solved by the column generation technique:
  - build the *Restricted Master Problem* (RMP), i.e. ($\underline{PBF}$), in which the full set of routes R is replaced by a (much) smaller subset $\mathcal{R} \subset R$;
  - initialise $\mathcal{R}$;
  - (RMP) is solved by any algorithm for Linear Programming, which also solves its dual;

2 - Price-and-Branch for the path-based formulation

With ξ_v^h, π_h^t, ρ_v^{h+}, ρ_v^{h-}, σ_h^I, σ_h^E being the dual variables of (2), (3), (22), (23), (24) and (25), respectively, the dual of the (RMP) is:

$$\max \ \sum_{h\in H}\Big(\sum_{v\in V}\big(\xi_v^h d_v^{1:h} - \rho_v^{h+}u_v^{h+} - \rho_v^{h-}u_v^{h-}\big) - \sigma_h^I u_h^I - \sigma_h^E u_h^E - \sum_{t\in T}\pi_h^t k_h^t \Big) \qquad (10)$$

$$s.t. \ \sum_{\mathbf{h}\in\mathbf{H}}\sum_{\mathbf{v}\in\mathbf{V}} \xi_{\mathbf{v}}^{\mathbf{h}}\alpha_{\mathbf{v},\mathbf{r}} - \pi_{\mathbf{h(r)}}^{\mathbf{t(r)}} \leq \mathbf{c_r} \qquad r\in\mathcal{R} \qquad (11)$$

$$\xi_v^h - \rho_v^{h+} - \sigma_h^I + \sigma_h^E \leq f_v^{h+} \qquad v\in V \ , \ h\in H \qquad (12)$$

$$-\xi_v^h - \rho_v^{h-} + \sigma_h^I - \sigma_h^E \leq f_v^{h-} \qquad v\in V \ , \ h\in H \qquad (13)$$

$$\rho_v^{h+} \ , \ \rho_v^{h-} \in \mathbb{R}_+ \qquad v\in V \ , \ h\in H \qquad (14)$$

$$\pi_h^t \in \mathbb{R}_+ \qquad t\in T \ , \ h\in H \qquad (15)$$

$$\sigma_h^I \ , \ \sigma_h^E \in \mathbb{R}_+ \qquad h\in H \qquad (16)$$

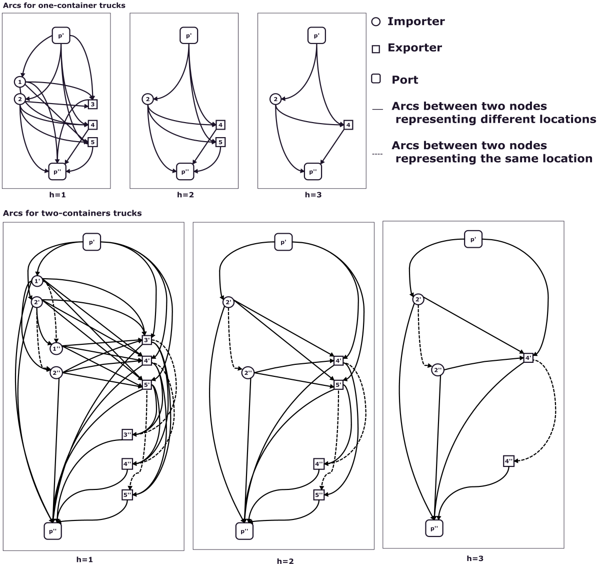
Constraints (11) correspond to each route in the **restricted** subset $\mathcal{R}$ of routes;

2 - Price-and-Branch for the path-based formulation

- If the dual solution of (RMP) satisfies constraints (11) for **all** feasible routes R, $(\underline{PBF})$ is optimally solved, else determine a route $\bar{r}$ with negative *reduced cost*: $c^*_{\bar{r}} = c_{\bar{r}} - \sum_{h \in H} \sum_{v \in V} \xi^{*h}_{v} \alpha_{v,\bar{r}} + (\pi^{t(\bar{r})}_{h(\bar{r})})^*$;
- Discarding the last (constant) term, $c^*_{\bar{r}}$ is the sum of the reduced costs of the arcs of the cycle (comprised the self-loops), where the reduced cost of arc (i,j) is $c^*_{ij} = c_{ij} - \xi^*_i$;
- Determining a route of negative reduced cost (or pricing problem) can be reduced to a collection of Shortest Path Problems (SPP) on tailor-made *acyclic* networks $\bar{G}^t_h = (\bar{N}^t_h, \bar{A}^t_h)$, one for each $t \in T$ and $h \in H$.
- These networks are obtained from the (physical) sub-graphs $G^t_h = (N_h, A^t_h)$ by "unrolling the self-loops".

2 - Price-and-Branch for the path-based formulation

Acyclic "step-expanded" networks:



2 - Price-and-Branch for the path-based formulation

- Efficient acyclic SPP algorithms to solve the pricing problem;
- No negative cost cycle can ever form;
- Quite good bounds by column generation, but in general no integer feasible solution;
- Price-and-Branch (*P&B*): pass the final set of routes $\mathcal{R}$ of (*RMP*) to a general-purpose MILP solver and solve the (small-ish) program to integer optimality;
- *P&B* quite effective and efficient when the root node gap is low.

3 - Compact arc-flow formulation

- Use the pricing problem to construct a "compact" (flow-based) formulation to the entire problem with the same strong bound as the (PBF);
- Expand the former step-expanded networks $\bar{G}_h^t$ by adding the single "return arc" (p'', p'); all cycles necessarily use this "return arc";
- For each arc $(i, j) \in \bar{A}_h^t$, x_{ij}^{th} is the number of trucks of type t doing that particular leg (comprised the "no-travel arcs" (v', v'') for some customer $v \in V_h$ at time period h) with unitary cost C_{ij}^t.

3 - Compact arc-flow formulation

Let FS_h^t and BS_h^t be the *forward star* and *backward star* of a node. The Step-Expanded Arc-Flow Formulation (*SEAF*) is:

$$\min \sum_{\mathbf{h}\in\mathbf{H}} \sum_{\mathbf{t}\in\mathbf{T}} \sum_{(\mathbf{i},\mathbf{j})\in\bar{\mathbf{A}}_\mathbf{h}^\mathbf{t}} \mathbf{c}_{\mathbf{ij}}^\mathbf{t}\mathbf{x}_{\mathbf{ij}}^{\mathbf{th}} + \sum_{h\in H}\sum_{v\in V}(f_v^{h+}s_v^{h+} + f_v^{h-}s_v^{h-}) \tag{17}$$

$$s.t. \sum_{(\mathbf{j},\mathbf{i})\in\mathbf{BS}_\mathbf{h}^\mathbf{t}(\mathbf{i})} \mathbf{x}_{\mathbf{ji}}^{\mathbf{th}} - \sum_{(\mathbf{i},\mathbf{j})\in\mathbf{FS}_\mathbf{h}^\mathbf{t}(\mathbf{i})} \mathbf{x}_{\mathbf{ij}}^{\mathbf{th}} = \mathbf{0} \qquad \mathbf{i}\in\bar{\mathbf{N}}_\mathbf{h}^\mathbf{t} \;,\; \mathbf{t}\in\mathbf{T} \;,\; \mathbf{h}\in\mathbf{H} \tag{18}$$

$$\sum_{\mathbf{k}=1,\dots,\mathbf{h}} \left(\sum_{(\mathbf{j},\mathbf{v})\in\mathbf{BS}_\mathbf{k}^1(\mathbf{v})} \mathbf{x}_{\mathbf{jv}}^{1\mathbf{k}} + \sum_{(\mathbf{j},\mathbf{v}')\in\mathbf{BS}_\mathbf{k}^2(\mathbf{v}')} \mathbf{x}_{\mathbf{jv}'}^{2\mathbf{k}} + \sum_{(\mathbf{j},\mathbf{v}'')\in\mathbf{BS}_\mathbf{k}^2(\mathbf{v}'')} \mathbf{x}_{\mathbf{jv}''}^{2\mathbf{k}} \right)$$

$$+ s_v^{h+} - s_v^{h-} = d_v^{1:h} \qquad v\in V \;,\; h\in H \tag{19}$$

$$\mathbf{x}_{\mathbf{p}''\mathbf{p}'}^{\mathbf{th}} \le \mathbf{k}_\mathbf{h}^\mathbf{t} \qquad \mathbf{t}\in\mathbf{T} \;,\; \mathbf{h}\in\mathbf{H} \tag{20}$$

$$\mathbf{x}_{\mathbf{ij}}^{\mathbf{th}} \in \mathbb{N} \qquad (\mathbf{i},\mathbf{j})\in\bar{\mathbf{A}}_\mathbf{h}^\mathbf{t} \;,\; \mathbf{t}\in\mathbf{T} \;,\; \mathbf{h}\in\mathbf{H} \tag{21}$$

$$s_v^{h+} \le u_v^{h+} \qquad v\in V \;,\; h\in H \tag{22}$$

$$s_v^{h-} \le u_v^{h-} \qquad v\in V \;,\; h\in H \tag{23}$$

$$\sum_{v\in I} s_v^{h+} - \sum_{v\in I} s_v^{h-} \le u_h^I \qquad h\in H \tag{24}$$

$$\sum_{v\in E} s_v^{h-} - \sum_{v\in E} s_v^{h+} \le u_h^E \qquad h\in H \tag{25}$$

$$s_v^{h+} \;,\; s_v^{h-} \in \mathbb{R}_+ \qquad v\in V \;,\; h\in H \tag{26}$$

3 - Compact arc-flow formulation

- The LP relaxation of (*SEAF*) has the same lower bound as (*PBF*):
  - The Lagrangian relaxation w.r.t. constraints (19)–(20) decomposes into as many flow subproblems as the networks $\bar{G}_h^t$ plus as many univariate problems on slack variables.
  - By calling ξ_v^h and π_h^t respectively the Lagrangian multipliers of (19) and (20), the costs of the Lagrangian Relaxation are the reduced costs of the former pricing problem; the optimal value of the slack variables in the subproblems is null;
  - The solution of the Lagrangian relaxation actually reduces to precisely the same acyclic SPPs between p' and p'' as the pricing problem for (*PBF*);
  - Lagrangian (network) subproblems obviously have integrality property, hence (*SEAF*) is "as tight" as (*PBF*).

Experimentation

- The size of (*SEAF*) grows about quadratically in the number of customers rather than quartically as the (*PBF*);
- The number of final routes at the end of the column generation can be much smaller than the size of (*SEAF*);

- Test to what extent the former formulations can be solved in a restricted but realistic problem and in the general problem;
- Analyse how increased customer flexibility levels affect routing costs (e.g. how to size potential incentives for flexibility).

- Setting:
  - Cplex 12.8 on a 3.00 GHz processor, 16 GB of RAM;
  - Maximum running time of 3 hours;
  - required relative gap $= 0.01\%$.

The restricted problem

- Customers only specify a total demand over all time periods, rather than a daily desired demand;
- Three types of customers in terms of *flexibility level*:
  - *no-flexibility* customers, who require to be served only in a desired day;
  - *medium-flexibility* customers, who accept to be served in two consecutive days of the planning horizon;
  - *high-flexibility* customers, who accept to be served in any day.
- No penalties for earlier/later than desired services within flexibility periods (i.e. the demand of customers can be freely subdivided between these periods);
- No capacity on the maximum number of container loads that can be served early or late;
- No capacities and costs for containers left at the port for late delivery and early collection.

Tests on the restricted problem (small instances)

				PBF-r			*P&B*						*SEAF-r*					
I	E	k_1^h	k_2^h	R	τ_n	t_n	it	B^*	t_{PB}	t_M	t_P	t_{lm}	t_{NA}	τ_{NA}	cut	gap_B	gap_A	gap
2	48	22	56	16278	1.02	0.56	37	214	11.1	9.64	1.22	0.50	0.31	0.08	52	1.17	0.00	0.55
5	45	21	54	62850	4.19	1.67	38	217	5.58	3.92	1.39	0.30	0.27	0.13	52	1.18	0.00	1.56
10	40	18	50	177750	11.02	8.01	41	231	6.74	5.07	1.38	0.15	0.25	0.36	67	1.65	0.00	0.60
15	35	17	42	295500	17.97	14.05	41	259	7.38	5.61	1.42	0.18	0.47	0.81	50	0.80	0.00	0.68
20	30	13	37	379350	23.13	23.23	56	301	10.2	8.10	1.73	0.20	0.33	0.67	46	1.25	0.01	0.75
25	25	11	32	407550	24.79	16.23	51	302	8.28	6.22	1.63	0.11	0.30	0.46	37	1.64	0.00	0.79
30	20	12	32	563670	22.27	16.23	47	304	8.79	6.74	1.60	0.13	0.48	0.45	42	1.17	0.00	0.91
35	15	15	39	285000	17.43	13.02	48	285	10.1	7.95	1.77	0.19	0.80	0.35	51	1.49	0.00	0.79
40	10	17	46	165750	9.96	4.61	46	272	7.89	5.94	1.53	0.17	0.39	0.52	39	0.52	0.00	0.74
45	5	20	50	53850	3.39	2.01	41	236	8.19	5.95	1.84	0.27	0.42	0.88	37	0.98	0.00	0.65
48	2	22	55	11862	0.73	1.14	39	226	5.67	4.08	1.31	0.24	0.39	0.41	72	0.59	0.00	1.52
2	48	0	68	16278	1.02	1.17	37	310	5.95	4.30	1.24	0.24	0.36	0.08	12	1.39	0.00	1.36
5	45	0	65	62850	4.19	2.71	39	308	6.52	4.76	1.35	0.26	0.36	0.13	54	1.19	0.00	0.55
10	40	0	60	177750	11.02	5.11	45	323	8.93	6.67	1.76	0.26	0.36	0.29	67	0.71	0.00	0.47
15	35	0	51	295500	17.97	14.91	42	289	6.41	4.66	1.37	0.20	0.42	0.79	57	1.52	0.00	1.07
20	30	0	44	379350	23.13	13.29	48	319	7.66	5.63	1.58	0.17	0.31	0.56	29	1.21	0.00	1.20
25	25	0	38	407550	24.79	17.49	43	303	7.16	5.36	1.38	0.10	0.36	0.31	24	1.66	0.00	0.71
30	20	0	38	563670	22.27	16.75	46	313	7.37	5.40	1.49	0.18	0.27	0.67	46	1.07	0.00	1.09
35	15	0	47	285000	17.43	7.92	48	329	7.59	5.56	1.56	0.39	0.33	0.99	46	0.88	0.00	1.79
40	10	0	55	165750	9.96	3.81	45	307	6.92	5.11	1.41	0.81	0.38	0.70	47	1.63	0.00	1.27
45	5	0	60	53850	3.39	1.84	42	323	6.99	5.15	1.39	0.25	0.36	0.83	57	0.93	0.00	1.51
48	2	0	66	11862	0.73	1.23	39	317	5.93	4.17	1.34	0.24	0.31	0.41	51	1.47	0.00	1.33

Instances adapted from Lai et al. (2013)

Tests on the restricted problem (average instances)

	I	E	k_1^h	k_2^h	PBF-r R	τ_n	t_n	P&B it	B^*	t_{PB}	t_M	t_P	t_{lm}	SEAF-r t_{NA}	τ_{NA}	cut	gap_B	gap_A	gap
A	20	5	4	56	2.1e+4	1.36	0.43	18	116	2.16	1.88	0.19	0.09	0.16	0.83	19	2.45	0.00	3.04
B	20	10	3	51	8.5e+4	2.33	1.86	27	144	2.93	2.40	0.36	0.16	0.34	0.82	26	1.92	0.00	4.18
C	20	20	4	47	3.3e+5	9.62	2.24	35	210	5.01	4.17	0.68	0.13	0.61	0.90	35	2.44	0.00	5.75
D	30	8	7	74	1.2e+5	6.12	9.13	24	158	3.05	2.45	0.45	0.13	0.49	0.59	76	1.78	0.00	4.28
E	30	15	6	69	3.5e+6	7.90	14.1	31	204	4.39	3.54	0.72	0.11	0.72	0.56	69	3.31	0.00	4.67
F	30	30	7	79	1.6e+6	18.01	211	55	328	9.46	7.13	1.87	0.26	0.77	0.52	267	2.44	0.00	5.81
G	45	12	8	112	4.9e+5	10.52	31.5	35	248	5.52	4.05	1.15	0.14	0.62	0.29	49	1.66	0.00	5.52
H	45	23	6	97	1.8e+6	16.83	-	55	308	9.54	6.99	2.12	0.18	1.18	0.84	239	2.22	0.00	6.55
I	45	45	9	129	8.3e+6	37.89	-	81	479	22.9	16.1	5.80	1.00	2.29	0.88	181	1.60	0.00	4.82
J	75	19	12	194	4.1e+6	23.63	-	62	423	13.4	8.80	3.92	0.29	3.82	0.63	188	1.75	0.01	5.81
K	75	38	13	177	1.6e+7	55.3	-	85	513	20.8	12.8	6.89	0.20	4.59	0.81	128	1.46	0.00	5.52
L	75	75	14	202	6.3e+7	190.2	-	140	836	46.8	27.0	17.1	0.42	5.19	0.62	183	1.70	0.00	6.69
M	100	25	17	236	1.2e+7	42.12	-	77	542	19.9	11.4	7.36	0.22	4.08	0.74	140	1.47	0.00	3.22
N	100	50	21	258	4.9e+7	139.9	-	97	660	29.4	15.6	12.1	0.23	5.09	0.88	216	1.33	0.00	4.44

Instances adapted from Goetschalckx and Jacobs-Blecha (1989)

Tests on the restricted problem (large instances)

				PBF-r	P&B						SEAF-r										
I	E	k_1^h	k_2^h	R	it	B^*	t_{PB}	t_M	t_P	t_{lm}	t_{NA}	τ_{NA}	cut	N	gap_B	gap_A	t_r	gap_r	t_O	gap_O	gap
72	37	55	494	1.5e+7	79	480	18.7	11.5	6.23	0.4	5.3	0.4	126	0	1.17	0.00	5	0.00	5.3	-	2.3
108	20	56	503	9.9e+6	80	517	32.9	17.5	13.5	0.7	90	0.6	0	1214	0.00	0.00	12	0.33	35	-	4.5
103	44	54	489	3.9e+7	111	696	50.0	23.8	25.0	0.2	11	0.8	167	0	0.80	0.00	11	0.00	11	-	5.0
192	16	143	1283	1.7e+7	104	751	51.1	31.0	17.1	0.1	78	1.2	176	3432	0.40	0.02	10	0.09	78	-	3.4
75	175	131	1177	3.5e+8	201	1150	126	46.4	79.0	0.4	81	1.4	492	1352	0.47	0.04	9	0.64	39	-	4.0
28	298	225	2016	1.2e+8	251	1108	142	38.7	100	0.3	137	1.7	585	1782	0.45	0.02	18	0.77	137	-	3.7
196	196	146	1311	2.7e+9	557	2640	681	220	458	1.6	1231	1.9	739	3854	0.39	0.05	18	0.19	1181	-	4.3
144	335	251	2256	4.5e+9	401	2215	572	126	444	0.3	752	2.1	779	1810	0.30	0.03	34	0.35	752	-	3.4
258	254	147	1316	8.3e+9	645	3223	1021	262	756	0.8	924	2.3	1030	1952	0.55	0.04	52	0.37	924	-	6.1
392	168	218	1954	8.5e+9	461	2496	779	152	623	0.7	10800	2.5	739	5195	0.55	0.06	63	0.41	6483	0.01	4.9
500	85	380	341	3.3e+9	425	2589	828	136	690	0.5	3531	2.3	858	49016	0.22	0.04	60	0.20	3421	-	3.7
438	188	329	2958	1.3e+10	566	3083	1173	219	951	0.6	10800	2.4	637	9424	0.23	0.02	87	0.23	7359	0.02	3.5
490	210	271	2437	1.9e+10	614	3507	1532	262	1266	0.8	10800	2.8	818	1786	0.24	0.02	127	0.39	5458	0.01	5.1
251	585	432	3881	4.1e+10	1682	8208	10800	5174	5624	1.8	10800	2.9	704	1765	0.30	0.07	249	—	7528	0.06	3.9
140	775	579	5205	2.1e+10	1206	6034	10800	4161	6637	1.1	10800	3.2	469	1769	0.23	0.06	362	—	9452	0.06	4.0
500	500	276	2482	1.2e+11	2364	11763	10800	4439	6358	2.1	10800	3.5	661	1775	0.34	0.08	534	—	8297	0.07	6.9

Instances adapted from Uchoa et al. (2017)

Tests on the general problem (small instances)

				PBF			SEAF				
I	E	k_1^h	k_2^h	R	τ_n	t_n	t_{NA}	τ_{NA}	cut	gap_B	gap_A
2	48	22	56	16278	1.02	0.39	0.11	0.31	54	1.17	0.00
5	45	21	54	62850	4.19	1.67	0.16	0.27	52	1.18	0.00
10	40	18	50	177750	11.0	8.00	0.20	0.25	62	1.65	0.00
15	35	17	42	295500	17.9	13.9	0.36	0.47	53	0.80	0.00
20	30	13	37	379350	23.1	19.4	0.19	0.33	47	1.25	0.00
25	25	11	32	407550	24.7	16.2	0.33	0.30	32	1.64	0.00
30	20	12	32	563670	22.2	16.2	0.37	0.48	48	1.17	0.00
35	15	15	39	285000	17.4	13.0	0.47	0.80	54	1.49	0.00
40	10	17	46	165750	9.96	4.61	0.20	0.39	46	0.52	0.00
45	5	20	50	53850	3.39	2.01	0.30	0.42	41	0.98	0.00
48	2	22	55	11862	0.73	1.14	0.27	0.39	76	0.59	0.00
2	48	0	68	16278	1.02	1.17	0.35	0.36	18	1.39	0.00
5	45	0	65	62850	4.19	2.71	0.36	0.36	59	1.19	0.00
10	40	0	60	177750	11.0	5.11	0.37	0.36	63	0.71	0.00
15	35	0	51	295500	17.9	14.53	0.40	0.42	51	1.52	0.00
20	30	0	44	379350	23.1	13.11	0.32	0.31	30	1.21	0.00
25	25	0	38	407550	24.7	14.02	0.39	0.36	31	1.66	0.00
30	20	0	38	563670	22.2	13.64	0.16	0.27	34	1.07	0.00
35	15	0	47	285000	17.4	7.92	0.31	0.33	49	0.88	0.00
40	10	0	55	165750	9.96	3.81	0.33	0.38	50	1.63	0.00
45	5	0	60	53850	3.39	1.84	0.17	0.36	54	0.93	0.00
48	2	0	66	11862	0.73	1.23	0.19	0.31	56	1.47	0.00

Instances adapted from Lai et al. (2013)

Tests on the general problem (average instances)

					PBF			SEAF				
	I	E	k_1^h	k_2^h	R	τ_n	t_n	t_{NA}	τ_{NA}	cut	gap_B	gap_A
A	20	5	4	56	2.1e+4	1.36	0.33	0.05	0.83	14	0.60	0.00
B	20	10	3	51	8.5e+4	2.33	1.73	0.19	0.82	26	0.69	0.00
C	20	20	4	47	3.3e+5	9.62	2.00	0.26	0.90	42	0.73	0.00
D	30	8	7	74	1.2e+5	6.12	7.93	0.44	0.59	79	0.59	0.00
E	30	15	6	69	3.5e+6	7.90	10.3	0.20	0.56	72	0.92	0.00
F	30	30	7	79	1.6e+6	18.0	183	0.48	0.52	144	0.19	0.00
G	45	12	8	112	4.9e+5	10.5	33.1	0.59	0.29	60	0.83	0.00
H	45	23	6	97	1.8e+6	16.8	66.6	0.65	0.84	200	1.18	0.00
I	45	45	9	129	8.3e+6	37.8	271	0.83	0.88	189	0.66	0.00
J	75	19	12	194	4.1e+6	23.6	184	0.74	0.63	201	0.91	0.00
K	75	38	13	177	1.6e+7	55.3	211	0.98	0.81	146	0.95	0.00
L	75	75	14	202	6.3e+7	190	488	2.11	0.62	193	1.03	0.00
M	100	25	17	236	1.2e+7	42.1	369	1.50	0.74	154	1.19	0.00
N	100	50	21	258	4.9e+7	139	375	1.13	0.88	102	0.04	0.01

Instances adapted from Goetschalckx and Jacobs-Blecha (1989)

Tests on the general problem (large instances)

				PBF	SEAF							
I	E	k_1^h	k_2^h	R	t_{NA}	τ_{NA}	cut	N	gap_B	gap_A	t_r	gap_r
72	37	55	494	1.5e+7	3.4	0.42	184	0	0.88	0.00	3.48	0.00
108	20	56	503	9.9e+6	6.9	0.66	209	1708	1.56	0.01	1.33	0.97
103	44	54	489	3.9e+7	7.6	0.83	113	3381	0.70	0.02	1.48	0.20
192	16	143	1283	1.7e+7	53	1.23	192	3198	0.37	0.02	3.75	0.58
75	175	131	1177	3.5e+8	64	1.49	276	2146	0.29	0.02	4.10	1.31
28	298	225	2016	1.2e+8	17	1.72	484	1928	0.36	0.02	18.1	0.17
196	196	146	1311	2.7e+9	183	1.93	1247	3792	0.26	0.01	10.3	0.87
144	335	251	2256	4.5e+9	429	2.11	1283	3901	0.25	0.02	11.6	1.35
258	254	147	1316	8.3e+9	536	2.44	1304	16588	0.31	0.01	13.9	1.11
392	168	218	1954	8.5e+9	263	2.66	1202	1803	0.34	0.01	13.7	0.40
500	85	380	341	3.3e+9	270	2.36	974	1755	0.11	0.01	17.5	0.38
438	188	329	2958	1.3e+10	338	2.50	1334	5598	0.15	0.01	21.2	0.52
490	210	271	2437	1.9e+10	612	2.81	1477	9525	0.22	0.01	21.2	0.39
251	585	432	3881	4.1e+10	1929	2.97	874	41456	0.16	0.02	35.9	6.44
140	775	579	5205	2.1e+10	2526	3.82	3927	7609	0.17	0.06	56.1	9.97
500	500	276	2482	1.2e+11	2050	3.77	924	6684	0.22	0.01	514	—

Instances adapted from Uchoa et al. (2017)

Flexibility levels and routing costs

The larger the flexibility, the larger the savings in routing costs;

This effect may be counter-balanced by the penalties;

Results on medium-sized instances by the (*SEAF-r*) and (*SEAF*);

9 flexibility configurations:

- F_0: all customers have no flexibility;
- F_1: 25% of medium-flexibility customers, 75% of inflexible customers;
- F_2: 50% of medium-flexibility customers, 50% of inflexible customers;
- F_3: 75% of medium-flexibility customers, 25% of inflexible customers;
- F_4: all customers have medium flexibility;
- F_5: 25% of high-flexibility customers, 75% of medium-flexibility customer;
- F_6: 50% of high-flexibility customers, 50% of medium-flexibility customer;
- F_7: 75% of high-flexibility customers, 25% of medium-flexibility customer;
- F_8: all customers have high flexibility.

Flexibility levels and routing costs

(*SEAF-r*) vs (*SEAF*) on the former configurations;

(*SEAF*) with 3 settings of penalties f_v^{h-} and f_v^{h+}: 10% (low), 20% (medium) and 50% (high) of the cost of a direct trip to serve customer v by a two-container truck;

Average saving among all medium-sized instances:

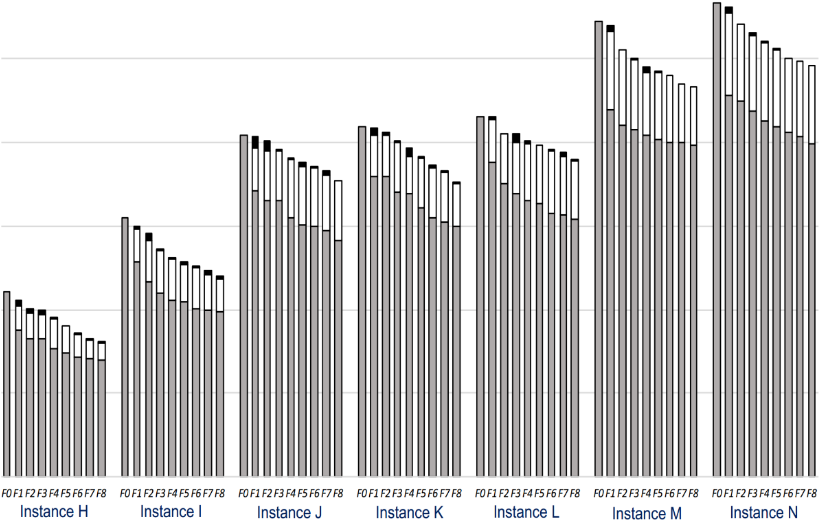
	F_0, F_1	F_1, F_2	F_2, F_3	F_3, F_4	F_4, F_5	F_5, F_6	F_6, F_7	F_7, F_8	Average	F_0, F_8
(*SEAF-r*)	2.58	4.60	3.88	4.63	3.55	4.00	3.93	3.56	3.84	26.25
(*SEAF*) 10%	4.95	4.95	2.98	4.52	3.31	3.84	3.71	3.67	3.99	26.84
(*SEAF*) 20%	1.66	1.52	1.90	2.01	2.08	1.66	0.56	1.75	1.64	12.70
(*SEAF*) 50%	0.12	0.07	0.25	0.12	0.52	0.29	0.50	0.16	0.25	2.23

(*SEAF-r*): Flexibility decrease routing costs by 26.25%!!!

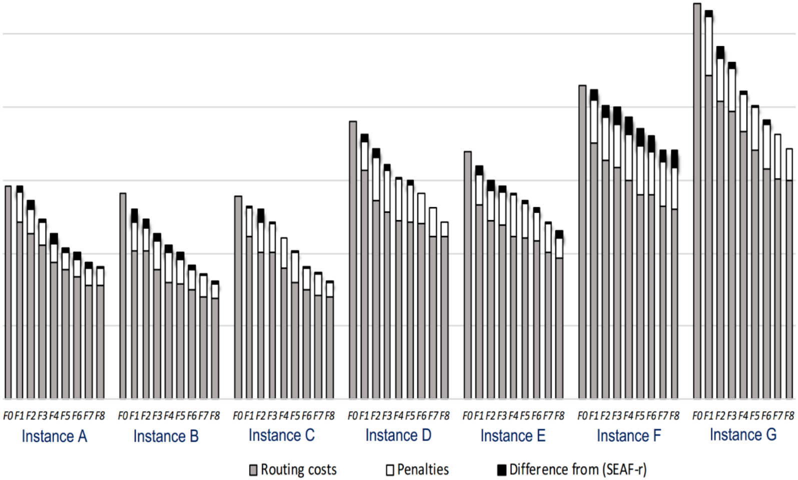
(*SEAF*): Costs can decrease further, but penalties need to be set "not too high".

Flexibility levels and routing costs

Small increase in flexibility means small decrease in costs for the carrier and high benefits for customers



Flexibility levels and routing costs



Conclusion

Key message:

- Choose strong formulations formulations of discrete optimization problems to solve them effectively;

Specific results:

- (*SEAF*) is the best formulation for this new drayage problem;
- Quantification of the possible savings by convincing customers to take a more flexible stance about service time;

Future research:

- (*SEAF*) by structured versions of the Dantzig-Wolfe decomposition algorithm;
- A richer problem setting.

Thank you for your attention.
Questions?